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Packet Size Sequence Modeling of Reliable Transmission Window Protocols over Links with Bernoulli Bit-Errors

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Abstract

In this paper, we propose a Markov model that represents size-sequences of two kinds of packets for networks supporting reliable transmission window protocols such as TCP: 1) packets generated from messages by a sender at the original transmission (namely, *generated* packets) and 2) packets contained in frames (i.e., data-link level PDUs) transferred over data-links (referred to as *transferred* packets). This analytical model makes it possible to discuss the effect of the *retransmitted packet size preservation (RPSP)* property, which means that all sizes of transferred packets with the same sequence number at retransmissions are equal to that of the original transmission (identical to the generated packet-size). The effect of RPSP is noticeable in networks where the size distribution of generated packets has a considerable variation and transferred packets are frequently lost with a rate that depends on the packet size as in a network environment with bit-errors. Hence, we present analytical expressions of the mean generated packet size taking message-segmentation function into consideration and of the mean transferred packet size representing RPSP in the environment where bit-errors occur according to the Bernoulli bit-error model. Furthermore, we demonstrate numerical results when message sizes are exponentially distributed, where explicit expressions can be obtained for various different bit-error rates, retransmission schemes, window sizes, and payload sizes. Among the key findings is the the fact that RPSP cannot be negligible in the following case, where 1) selective retransmission or go-back-N retransmission with small window sizes is performed, 2) the message-segmentation occurrence probability is relatively small, and 3) the bit error rate is high (e.g., 1×10^{-4} that is the mean bit-error rate of a wireless link in an industrial environment).

Key words – TCP, Reliable transmission window protocol, Packet size sequence, Bernoulli bit-errors, Retransmitted packet size preservation property, Message-segmentation

I. Introduction

Data transfers over the Internet suffer from corruption of protocol data units (PDUs) due to bit-errors and losses caused by congestion. To provide an error-free transmission service for reliable applications like the transfer of the Web pages by HTTP (hypertext transfer protocol) over such networks, it is necessary for each station (or host) to use communication protocols that include flow-control and error-recovery functions. For example, each station implements one or more reliable transmission protocols based on the sliding-window mechanism (referred to as reliable transmission window protocols: RWPs[1]).

The packets (i.e., RWP-layer PDUs) corrupted/lost within the network are retransmitted by the error-recovery function specified in the RWP. In general, such retransmitted packets with the same sequence number are equal in size to the packet in the original transmission. We call this property retransmitted packet size preservation: RPSP. The effect of RPSP on RWP performance will appear in some cases. In particular, this effect clearly exists when the transferred packets, which are the packets contained in frames transferred over data-links, are frequently lost

[1]Typical data link protocols categorized into RWPs are HDLC (high-level data link control) [1], IrLAP (infrared link access protocol) [2] and LLC2 (logical link control type 2) [3], and a typical transport protocol is TCP-Reno/NewReno/SACK [4]–[6].

with a rate that depends on the packet size, as in wireless link environments, and the size distribution of generated packets, which are packets generated from a message at the original transmission, has a considerable variation, resulting in the distribution of transferred packet sizes being markedly different from that of generated packet sizes.

However, in previous work on RWP performance analysis over links with bit-errors, such as [7], [8] for HDLC-based protocols and [9]–[11] for TCP, the effect of RPSP was ignored. In [7], [8], the generated packet sizes are assumed to be constant, although message-sizes (and hence generated packet sizes) are revealed to have a significant variation (this can be seen for example in HTTP-traffic observations reported in [12]). On the other hand, for the performance model presented in [10], the transferred packet sizes are assumed to be independently distributed in each transferred packet (re)transmission, regardless of RPSP. In this paper, we analyze the size-sequence of transferred packets given by the distribution of the generated packet sizes, taking into account the effect of RPSP caused by the error-recovery function, for network environments interconnected by links with Bernoulli bit-errors.

Some RWPs such as TCP include a message-segmentation function that allows an RWP-sender to divide a message larger than the payload size (or maximum segment size in TCP/IP terminology) into multiple generated packets. This function imposes constraints on the generated packet size, weakens the effect of RPSP. Here, we propose a Markov model for the size-sequence of generated packet sizes that captures the behavior of the message segmentation.

The rest of the paper is organized as follows. In the next section, we describe the communication system model underlying our study. In Section III, the distribution and mean of the size-sequence of generated packets are derived from a given distribution of message sizes. Section IV derives the distribution and mean of the size-sequence of transferred packets using the generated packet size distribution. Section V demonstrates some numerical results when message-sizes are assumed to be exponentially distributed. In this case, explicit expressions of the mean sizes of the generated and transferred packets can be obtained. In this Section, we also investigate the effect of RPSP on the ratio of the mean size of transferred packets to that of generated packets for different RWP-retransmission schemes (i.e., selective retransmission and go-back-N retransmission), window sizes, and payload size versus bit-error rates. Section VI summarizes the paper and mentions future work.

II. Communication system model

In this section, we first explain the three-layered wireless communication system model under consideration. Next, the model of PDUs exchanged between peer entities at the same layer are described. Then, we discuss the ways to implement message-segmentation function. To model the effect of RPSP, we further explain the packet terminology introduced in this paper. Finally, we describe the frame-loss model adopted in this paper.

A. Layer model

We consider a communication system where two stations (a sender and a receiver) are interconnected through a link that experiences bit-errors. A conceptual representation of the communication system is shown in Fig. 1. Each station is constructed of three layers: an RWP layer (a layer carrying out a specified RWP), a higher layer (i.e., a layer including several RWP applications), and a lower layer pertaining to a wireless link driver.

B. PDU model

We define PDUs used throughout this paper as follows:

Messages: PDUs that are generated by a traffic source at the sending-side higher layer with a given size-distribution, and terminated by a corresponding sink at the receiving-side higher layer.

Packets: PDUs that are created from messages and transferred from the sender-side RWP-layer to the receiver-side RWP-layer (which corresponds to “data segments” if the RWP is assumed to be TCP). Whenever a packet is created, a sequence-number $\mathsf{seqNum} \geq 0$ is assigned[2], and recorded in the header at each packet.

Frames: PDUs that are made from packets and transferred over data-links.

[2]TCP counts the packet being sent and acknowledged by its byte number not its packet number. In this paper, seqNum is assumed to be represented in terms of packets.

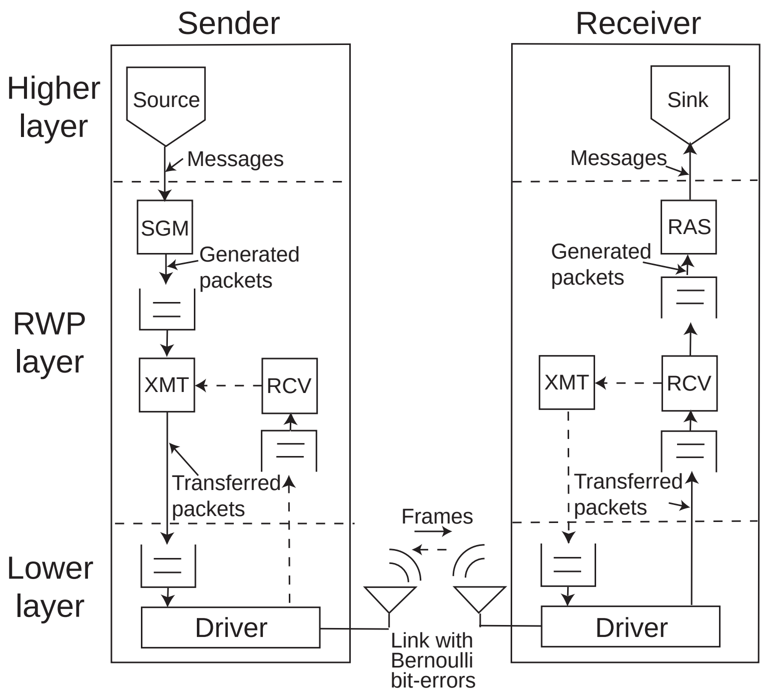

Fig. 1. Communication system model.

(Note: symbols SGM and RAS denote message segmentation and reassembly functions, respectively. Symbols XMT and RCV represents functions for sending and receiving PDUs, respectively. Arrows $\rightarrow$ and $\dashrightarrow$ represent flows of PDUs carrying higher layer data, i.e., a message, and control information, such as ACK, respectively.)

C. Message-segmentation model

Size of packet which can be transmitted is limited to the payload size ℓ_d (or maximum segment size in TCP/IP terminology). Therefore, some RWPs that include a message-segmentation function, such as TCP, allow an RWP-sender to divide a single message into multiple generated packets if the message size is greater than ℓ_d. There are two ways to implement the message-segmentation function:

- **Message arrival basis segmentation scheme (MAS)**: When a message arrives at the RWP-layer, it is divided into multiple packets if its size exceeds ℓ_d. Then, they are stored in the associated send buffer.
- **Packet (re)transmission basis segmentation scheme (PTS)**: When the RWP-layer (re-)sends a packet to the lower layer, it creates the packet which cannot exceed ℓ_d, from the messages stored in the associated send buffer (such as a socket buffer [13]).

For the RWPs that identify the packet being sent and acknowledged by its packet number not its byte number, MAS is employed. On the other hand, the RWPs which provide a byte-stream transmission service (i.e., counts in terms of bytes rather than packets) such as TCP allow a sender to implement PTS [14]. According to PTS, the sizes of the retransmitted packets are always not equal to that at the previous transmissions (this is called “re-packetization” [15]).

However, the packet size behavior of MAS is identical to that of PTS except in the following case: when a packet has been lost and new messages arrive at an RWP-layer until the lost packet is retransmitted. This case might happen when packets contain stream-type PDUs (or interactive PDUs including Telnet and Rlogin PDUs) and the packet loss is recovered by error-recovery mechanism which requires much long loss detection time such as timeout recovery.

In this paper, we model the message segmentation behavior according to MAS, since the population of the interactive PDUs tends to be much smaller than bulk PDUs (HTTP, FTP and electronic mail PDUs) and from analytical tractability. With MAS, all sizes of the retransmitted packets with the same seqNum become equal (i.e., according to RPSP).

D. Packet model

To model the effect of RPSP explicitly, we introduce the following packet terms:

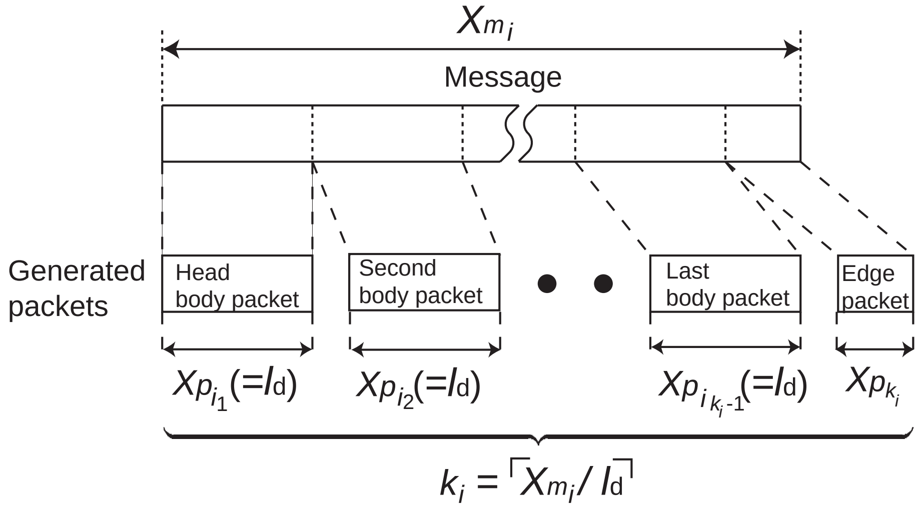

Fig. 2. Example of message-segmentation with MAS-scheme.

Generated packets: Packets that are generated from messages by a sender at the original transmission.
Transferred packets: Packets that are (re)transmitted encapsulated into frames, namely, packets transferred from an RWP-sender to an RWP-receiver.

Thus, due to RPSP, all the sizes of transferred packets with the same seqNum become equal to that of the generated packet.

E. Frame loss model

Frame-losses occur between a sender and a receiver mainly due to bit-error and congestion. In particular, the frame-loss probability due to bit-errors depends on the frame size. For example, assuming that bit-errors occur independently (i.e., according to the Bernoulli bit-error model) with bit error rate (BER) p_e, the transferred packet-corruption probability of the bit-length equal to x can be expressed as

$$p_b(x) = 1 - (1 - p_e)^{x+\ell_h}, \tag{1}$$

where ℓ_h is the total length of the control fields (or header and trailer) added by the RWP and lower layers[3]. We note that if bit-errors occur frequently, like in wireless communication environments, and the size distribution of the generated packets is non-deterministic, then the size distribution of the transferred packets becomes significantly different from it in some cases, as will be described in Section V. Hence, we focus our attention on the frame-losses due to bit-errors[4] occurring at the rate given in (1).

III. Modeling of generated packet size sequence

In this section, we first derive $F_{X_p}(\cdot)$, the stationary distribution function of a size-sequence of generated packets, given a distribution function of message-sizes $F_{X_m}(\cdot)$. Next, we calculate the mean ℓ_p of the generated packet size.

A. Derivation of stationary distribution of generated packet size sequence

Letting X_{m_i} denote the ith message size, we assume $\{X_{m_i}; i \in \mathcal{N}_0 \stackrel{\triangle}{=} \{0, 1, 2, \ldots\}\}$ is a sequence of mutually independent and identically distributed (*i.i.d.*) random variables with a common distribution function $F_{X_m}(\cdot)$ of mean message-size ℓ_m. Figure 2 illustrates an example of message-segmentation with MAS-scheme as discussed

[3]In wireless links, the bit-errors are reported to occur in bursts. As will be described in Section VI, the extension to the burst error model (i.e., correlated error model) is an open issue.

[4]When the transferred packets are lost due to congestion, $p_b(x)$ is generally independent of x. In this case, the stationary distribution of the transferred packets has a mean equal to that of the generated packets (see *Example 4*).

before. As shown in Fig. 2, if $X_{m_i} > \ell_d$, then the ith message is divided into multiple generated packets with a size-sequence $\{X_{p_{i_j}} : j = 1, \cdots, k_i\}$, where

$$\begin{cases} k_i &= \left\lceil \dfrac{X_{m_i}}{\ell_d} \right\rceil, \qquad\qquad \forall i \in \mathcal{N}_0, \\ X_{p_{i_j}} &= \begin{cases} \ell_d, & \text{for } j = 1, 2, \ldots, k_i - 1 \\ X_{m_i} - (k_i - 1)\ell_d, & \text{for } j = k_i. \end{cases} \end{cases} \tag{2}$$

Here $\lceil a \rceil$ represents the smallest integer that is greater than or equal to a. For the ith message, we refer to the $j\,(\leq k_i - 1)$ th generated packet as a "body"-packet and the last (i.e., k_i th) generated packet as an "edge"-packet (see Fig. 2). If $X_{m_i} \leq \ell_d$, the ith message is not segmented, and a single packet, which is identical to the original message, is generated. We also refer to this as an "edge"-packet, because it satisfies definition (2).

We constitute a stochastic process $\{X_{p_\kappa}; \kappa \in \mathcal{N}_0\}$, replacing an epoch label i_j by an in-sequence number $\kappa \in \mathcal{N}_0$ for $\{X_{p_{i_j}}\}$. Thus, κ represents the seqNum of the packet. To analyze the behavior of $\{X_{p_\kappa}\}$ through the framework of Markov chains, we introduce an auxiliary random variable Z_κ associated with X_{p_κ}. The random variable Z_κ is defined on the state space $S_Z = \{\mathrm{B}_1, \mathrm{B}_2, \cdots; \mathrm{E}_1, \mathrm{E}_2, \cdots\}$, where states B_r and E_s represent the rth body-packet and the edge-packet following $(s-1)$ body-packets, which are generated from each message, respectively. The stochastic process $\{Z_\kappa\}$ can be represented as a Markov chain having the following one-step transition probability matrix $\boldsymbol{P}_Z = [p_{Z_{\alpha\beta}}, \alpha \in S_Z, \beta \in S_Z]$ with entries

$$p_{Z_{\alpha\beta}} = \begin{cases} 1 - \dfrac{u_{r+1}}{u_r}, & \text{for } (\alpha, \beta) = (\mathrm{B}_r, \mathrm{E}_{r+1}) \text{ and } \forall r \in \mathcal{N} \\ \dfrac{u_{r+1}}{u_r}, & \text{for } (\alpha, \beta) = (\mathrm{B}_r, \mathrm{B}_{r+1}) \text{ and } \forall r \in \mathcal{N} \\ 1 - u_1, & \text{for } (\alpha, \beta) = (\mathrm{E}_s, \mathrm{E}_1) \text{ and } \forall s \in \mathcal{N} \\ u_1, & \text{for } (\alpha, \beta) = (\mathrm{E}_s, \mathrm{B}_1) \text{ and } \forall s \in \mathcal{N} \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\mathcal{N} \triangleq \{1, 2, \cdots\}$ and $u_r \triangleq \int_{r\ell_d}^{\infty} dF_{X_m}(x) = 1 - F_{X_m}(r\ell_d), r \in \mathcal{N}$. For a detailed derivation of (3), see APPENDIX I.

Let $F_{X_{p_\alpha}}(\cdot)$ be the conditional distribution function of X_{p_κ} when the state of Z_κ is $\alpha \in S_Z$. Then, we have:

$$\begin{aligned} F_{X_{p_{\mathrm{B}_r}}}(x) &\triangleq \Pr(X_{p_\kappa} \leq x \,|\, Z_\kappa = \mathrm{B}_r) \\ &= \mathbf{1}(x - \ell_d) = F_{X_{p_\mathrm{B}}}(x), \quad \forall r \in \mathcal{N}, \end{aligned} \tag{4}$$

$$\begin{aligned} F_{X_{p_{\mathrm{E}_s}}}(x) &\triangleq \Pr(X_{p_\kappa} \leq x \,|\, Z_\kappa = \mathrm{E}_s) \\ &= \frac{\Pr((s-1)\ell_d < X_{m_i} \leq (s-1)\ell_d + x)}{\Pr((s-1)\ell_d < X_{m_i} \leq s\ell_d)} \\ &= \begin{cases} 0, & x \leq 0 \\ \dfrac{F_{X_m}((s-1)\ell_d + x) - F_{X_m}((s-1)\ell_d)}{u_{s-1} - u_s}, & 0 < x \leq \ell_d \\ 1, & x > \ell_d \end{cases} \qquad \forall s \in \mathcal{N}. \end{aligned} \tag{5}$$

We find that the random variable X_{p_κ} is independent of another $X_{p_{\kappa'}}$ if Z_κ is given, due to the *i.i.d.* message size assumption and the definition of random variables $\{Z_\kappa\}$. Thus, we have

$$\begin{aligned} &\Pr\{X_{p_\kappa} \leq x_\kappa, X_{p_{\kappa'}} \leq x_{\kappa'}, Z_\kappa = z_\kappa, Z_{\kappa'} = z_{\kappa'} \,|Z_\kappa = z_\kappa, Z_{\kappa'} = z_{\kappa'}\} = \\ &\qquad\qquad \Pr\{X_{p_\kappa} \leq x_\kappa | Z_\kappa = z_\kappa\} \Pr\{X_{p_\kappa} \leq x_{\kappa'} | Z_\kappa = z_{\kappa'}\}. \end{aligned} \tag{6}$$

Consequently, the two-dimensional stochastic process $\{(X_{p_\kappa}, Z_\kappa) : \kappa \in \mathcal{N}_0\}$ forms another Markov chain. In the following, the two-dimensional Markov chain $\{(X_{p_\kappa}, Z_\kappa)\}$ is assumed to be in the steady state.

We denote the stationary-state probabilities for the Markov chain $\{Z_\kappa\}$ by π_{B_r} and π_{E_s}, respectively. They are given by

$$\pi_{\mathrm{B}_r} \triangleq \Pr(Z_\kappa = \mathrm{B}_r) = \Delta^{-1} u_r, \quad \forall r \in \mathcal{N}, \tag{7}$$

$$\pi_{\mathrm{E}_s} \triangleq \Pr(Z_\kappa = \mathrm{E}_s) = \Delta^{-1} (u_{s-1} - u_s), \quad \forall s \in \mathcal{N}, \tag{8}$$

where

$$\Delta \triangleq \sum_{s=0}^{\infty} u_s, \tag{9}$$

with $u_0 = 1$.

Let $F_{X_p}(\cdot)$ be the stationary distribution function of X_{p_κ}. The two-dimensional Markov chain $\{(X_{p_\kappa}, Z_\kappa)\}$ yields

$$\begin{aligned} F_{X_p}(x) &\triangleq \Pr(X_{p_\kappa} \le x) \\ &= \sum_{\alpha \in S_Z} \Pr(X_{p_\kappa} \le x | Z_\kappa = \alpha) \Pr(Z_\kappa = \alpha) \\ &= \sum_{r=1}^{\infty} \pi_{\mathrm{B}_r} F_{X_{p_{\mathrm{B}_r}}}(x) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} F_{X_{p_{\mathrm{E}_s}}}(x) \\ &= F_{X_{p_{\mathrm{B}}}}(x) \sum_{r=1}^{\infty} \pi_{\mathrm{B}_r} + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} F_{X_{p_{\mathrm{E}_s}}}(x), \\ &= (1 - \pi_{\mathrm{E}}) F_{X_{p_{\mathrm{B}}}}(x) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} F_{X_{p_{\mathrm{E}_s}}}(x), \end{aligned} \tag{10}$$

where the edge-packet occurrence probability π_{E} is given by

$$\pi_{\mathrm{E}} \triangleq 1 - \sum_{r=1}^{\infty} \pi_{\mathrm{B}_r} = \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s}. \tag{11}$$

From (8) and $\sum_{s=1}^{\infty} (u_{s-1} - u_s) = u_0 = 1$, (11) can be re-written as

$$\begin{aligned} \pi_{\mathrm{E}} &= \sum_{s=1}^{\infty} \Delta^{-1} (u_{s-1} - u_s) \\ &= \Delta^{-1} u_0 = \Delta^{-1}. \end{aligned} \tag{12}$$

B. Derivation of the mean generated packet size

From (5), (8) - (10), and (12), the mean size ℓ_p of a generated packet in the steady state can be written as

$$\begin{aligned} \ell_p &\triangleq E[X_{p_\kappa}] = \int_0^{\infty} x dF_{X_p}(x) \\ &= (1 - \pi_{\mathrm{E}}) \int_0^{\infty} x dF_{X_{p_{\mathrm{B}}}}(x) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} \int_0^{\infty} x dF_{X_{p_{\mathrm{E}_s}}}(x) \\ &= (1 - \pi_{\mathrm{E}}) \int_0^{\infty} x \delta(x - \ell_d) dx + \sum_{s=1}^{\infty} \frac{\pi_{\mathrm{E}_s}}{u_{s-1} - u_s} \int_{(s-1)\ell_d}^{s\ell_d} (x - (s-1)\ell_d) \, dF_{X_m}(x) \\ &= (1 - \pi_{\mathrm{E}}) \ell_d + \Delta^{-1} \sum_{s=1}^{\infty} \Big[\int_{(s-1)\ell_d}^{s\ell_d} x dF_{X_m}(x) - \ell_d (s-1)(u_{s-1} - u_s) \Big] \\ &= \pi_{\mathrm{E}} \ell_m, \end{aligned} \tag{13}$$

from

$$\sum_{s=1}^{\infty} \int_{(s-1)\ell_d}^{s\ell_d} x dF_{X_m}(x) = \int_0^{\infty} x dF_{X_m}(x) = \ell_m,$$

and

$$\sum_{s=1}^{\infty}(s-1)(u_{s-1}-u_s)=\sum_{s=1}^{\infty}u_s=\Delta-1.$$

Example 1: Assuming that the message-sizes are exponentially distributed with mean ℓ_m^e:

$$F^e_{X_m}(x)=1-e^{-\frac{x}{\ell_m^e}}. \tag{14}$$

In this case, we have

$$u_s=\int_{s\ell_d}^{\infty}dF_{X_m}(x)=e^{-\frac{s\ell_d}{\ell_m^e}}=u^{e\ s}, \tag{15}$$

with $u^e\triangleq u_1=e^{-\frac{\ell_d}{\ell_m^e}}$. Then, from (8), (10) and (11), the stationary distribution function $F^e_{X_p}(\cdot)$ is given by

$$\begin{aligned}F^e_{X_p}(x)&=(1-\pi^e_{\mathrm{E}})F_{X_{p_{\mathrm{B}}}}(x)+\sum_{s=1}^{\infty}\pi^e_{\mathrm{E}_s}F^e_{X_{p_{\mathrm{E}_s}}}(x)\\&=(1-\pi^e_{\mathrm{E}})F_{X_{p_{\mathrm{B}}}}(x)+\pi^e_{\mathrm{E}}F^e_{X_{p_{\mathrm{E}}}}(x),\end{aligned} \tag{16}$$

where

$$\pi^e_{\mathrm{E}_s}=(1-u^e)^2u^{e\ s-1}, \tag{17}$$

$$\pi^e_{\mathrm{E}}=\sum_{s=1}^{\infty}\pi^e_{\mathrm{E}_s}=1-u^e, \tag{18}$$

$$\begin{aligned}F^e_{X_{p_{\mathrm{E}_s}}}(x)&=\begin{cases}0, & x\le 0\\ \dfrac{1-e^{-\frac{x}{\ell_m^e}}}{1-u^e}, & 0<x\le\ell_d\\ 1, & x>\ell_d\end{cases}\\&\triangleq F^e_{X_{p_{\mathrm{E}}}}(x),\quad\forall s\in\mathcal{N}.\end{aligned} \tag{19}$$

We note that (19) shows that the size distributions of edge-packets $F^e_{X_{p_{\mathrm{E}_s}}}(x)$ for $s\in\mathcal{N}$ are independent of s because of the memoryless property of the exponential distribution.

Hence, from (13), the mean size of generated packets ℓ_p^e is given by

$$\ell_p^e=\pi^e_{\mathrm{E}}\ell_m^e=(1-u^e)\ell_m^e. \tag{20}$$

Clearly, (20) is verified in the following:

$$\ell_p^e=(1-\pi^e_{\mathrm{E}})\int_0^{\infty}xdF_{X_{p_{\mathrm{B}}}}+\pi^e_{\mathrm{E}}\int_0^{\infty}xdF^e_{X_{p_{\mathrm{E}}}}(x) \tag{21a}$$

$$=(1-\pi^e_{\mathrm{E}})\int_0^{\infty}x\delta(x-\ell_d)dx+\pi^e_{\mathrm{E}}\int_0^{\ell_d}xdF^e_{X_{p_{\mathrm{E}}}}(x) \tag{21b}$$

$$=(1-\pi^e_{\mathrm{E}})\ell_d+\pi^e_{\mathrm{E}}\int_0^{\ell_d}\frac{xe^{-\frac{x}{\ell_m^e}}}{1-u^e}dx \tag{21c}$$

$$=\text{Eq. (20)}. \tag{21d}$$

Remark 1: Letting σ_m^2 denote the variance of message size, the variance of generated packet size σ_p^2 is given by

$$\begin{aligned}
\sigma_p^2 &\triangleq E[X_{p_\kappa}^2] - E[X_{p_\kappa}]^2 \\
&= (1-\pi_{\mathrm{E}})dF_{X_{p_{\mathrm{B}}}}(x) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} \int_0^{\infty} x^2 dF_{X_{p_{\mathrm{E}_s}}}(x) - \ell_p^2 \\
&= (1-\pi_{\mathrm{E}}) \int_0^{\infty} x^2 \delta(x-\ell_d)dx + \sum_{s=1}^{\infty} \frac{\pi_{\mathrm{E}_s}}{u_{s-1}-u_s} \int_{(s-1)\ell_d}^{s\ell_d} (x-(s-1)\ell_d)^2 dF_{X_m}(x) - \ell_p^2 \\
&= (1-\pi_{\mathrm{E}})\ell_d^2 \\
&\quad + \Delta^{-1} \sum_{s=1}^{\infty} \left[\int_{(s-1)\ell_d}^{s\ell_d} x^2 dF_{X_m}(x) - 2\ell_d(s-1) \int_{(s-1)\ell_d}^{s\ell_d} x dF_{X_m}(x) + (s-1)^2 \ell_d^2 \int_{(s-1)\ell_d}^{s\ell_d} dF_{X_m}(x) \right] \\
&\quad - \ell_p^2 \\
&= \pi_{\mathrm{E}}(\ell_m^2 + \sigma_m^2) + 2\pi_{\mathrm{E}}\ell_m\ell_d - 2\pi_{\mathrm{E}}\ell_d \sum_{s=0}^{\infty} v_s + 2\pi_{\mathrm{E}}\ell_d^2 \sum_{s=1}^{\infty} s u_s - \ell_p^2 \\
&= \pi_{\mathrm{E}}(\ell_m^2 + \sigma_m^2) + 2\pi_{\mathrm{E}}\ell_m\ell_d - 2\pi_{\mathrm{E}}\ell_d \left(\sum_{s=0}^{\infty} v_s - \ell_d \sum_{s=1}^{\infty} s u_s \right) - \ell_p^2,
\end{aligned} \tag{22}$$

from

$$\begin{aligned}
\sum_{s=1}^{\infty} \int_{(s-1)\ell_d}^{s\ell_d} x^2 dF_{X_m}(x) &= \int_0^{\infty} x^2 dF_{X_m}(x) = \sigma_m^2 + \ell_m^2, \\
\sum_{s=1}^{\infty} (s-1) \int_{(s-1)\ell_d}^{s\ell_d} x dF_{X_m}(x) &= \sum_{s=0}^{\infty} v_s - \ell_m, \\
\sum_{s=1}^{\infty} (s-1)^2 \int_{(s-1)\ell_d}^{s\ell_d} dF_{X_m}(x) &= 2\sum_{s=0}^{\infty} s u_s - \Delta + 1,
\end{aligned}$$

with $v_s \triangleq \int_{s\ell_d}^{\infty} x dF_{X_m}(x)$.

Example 2: Let σ_p^{e2} denote the variance of generated packet size when message sizes are exponentially distributed with mean ℓ_m^e, given in (14). Then, we have

$$\sigma_p^{e2} = 2\pi_{\mathrm{E}}^e {\ell_m^e}^2 + 2\pi_{\mathrm{E}}^e \ell_m^e \ell_d - 2\ell_m^e \ell_d - {\pi_{\mathrm{E}}^e}^2 {\ell_m^e}^2, \tag{23}$$

from

$$\begin{aligned}
\sum_{s=0}^{\infty} v_s^e - \ell_d \sum_{s=1}^{\infty} s u^{es} &= \sum_{s=0}^{\infty} (\ell_d s u^{es} + \ell_m^e u^{es}) - \ell_d \sum_{s=1}^{\infty} s u^{es} \\
&= \ell_m^e \sum_{s=0}^{\infty} u^{es} = \frac{\ell_m^e}{\pi_{\mathrm{E}}^e},
\end{aligned} \tag{24}$$

with $v_s^e \triangleq \int_{s\ell_d}^{\infty} x dF_{X_m}^e(x) = \frac{1}{\ell_m^e} \int_{s\ell_d}^{\infty} x e^{-\frac{x}{\ell_m^e}} dx$.

Now we vary ℓ_d for a fixed ℓ_m^e. Then, by taking the derivative of ${\sigma_p^e}^2$ with respect to ℓ_d, we get

$$\begin{aligned}
\frac{d{\sigma_p^e}^2}{d\ell_d} &= 2{\ell_m^e}^2 \frac{d\pi_{\mathrm{E}}^e}{d\ell_d} + 2\ell_m^e \ell_d \frac{d\pi_{\mathrm{E}}^e}{d\ell_d} + 2\pi_{\mathrm{E}}^e \ell_m^e - 2\ell_m^e - 2\pi_{\mathrm{E}}^e {\ell_m^e}^2 \frac{d\pi_{\mathrm{E}}^e}{d\ell_d} \\
&= 2e^{-\frac{\ell_d}{\ell_m^e}} \left\{ \ell_d - \ell_m^e \left(1 - e^{-\frac{\ell_d}{\ell_m^e}} \right) \right\} \\
&\geq 2e^{-\frac{\ell_d}{\ell_m^e}} \left\{ \ell_d - \ell_m^e \cdot \frac{\ell_d}{\ell_m^e} \right\} = 0,
\end{aligned} \tag{25}$$

since $x \geq 1 - e^{-x}$ for any $x > 0$. Thus, the variance ${\sigma_p^e}^2$ is increasing in ℓ_d.

Next, we discuss the variance $\sigma_p^{e\,2}$ for two limit situations: when ℓ_d is enough small, namely $\ell_d \approx 0$ and when ℓ_d is enough large compared to ℓ_m^e, that is $\ell_d \gg \ell_m^e$. If $\ell_d \approx 0$, then

$$\sigma_p^{e\,2} = \frac{l_d^3}{3\ell_m^e} + O(\Big(\tfrac{\ell_d}{\ell_m^e}\Big)^4) \approx 0 \quad \text{if } \ell_d \approx 0, \tag{26}$$

since

$$\begin{cases} \pi_{\mathrm{E}}^e = \dfrac{\ell_d}{\ell_m^e} - \dfrac{1}{2}\left(\dfrac{\ell_d}{\ell_m^e}\right)^2 + \dfrac{1}{6}\left(\dfrac{\ell_d}{\ell_m^e}\right)^3 + O(\Big(\tfrac{\ell_d}{\ell_m^e}\Big)^4), \\ \pi_{\mathrm{E}}^{e\,2} = \left(\dfrac{\ell_d}{\ell_m^e}\right)^2 - \left(\dfrac{\ell_d}{\ell_m^e}\right)^3 + O(\Big(\tfrac{\ell_d}{\ell_m^e}\Big)^4). \end{cases} \tag{27}$$

Hence, in this case the variance is very small, implying that almost all generated packets are body packets. On the other hand, if $\ell_d \gg \ell_m^e$, then

$$\sigma_p^{e\,2} \approx \ell_m^{e\ 2} \quad \text{if } \ell_d \gg \ell_m^e, \tag{28}$$

since $\sigma_p^{e\,2} = \ell_m^{e\ 2} - 2u^e\ell_m^e\ell_d - u^{e2}\ell_m^{e\ 2}$ and $u^e \approx 0$ in this case. Equation (28) is intuitively verified since almost all generated packets correspond to the respective messages with size-variance equal to $\ell_m^{e\ 2}$.

IV. Modeling of transferred packet size sequence

Let X_{q_t} denote the transferred packet size of the $t(\geq 0)$th transmission by an RWP sender. In this section, we derive $F_{X_q}(\cdot)$, the stationary distribution function of a size-sequence of transferred packets $\{\hat{X}_{q_t}; t \in \mathcal{N}_0\}$, using the analytical expression of $F_{X_p}(\cdot)$ obtained in the preceding section, and we calculate the mean size ℓ_q of transferred packets.

A. Derivation of stationary distribution of transferred packet size sequence

For a sequence of transferred packets with seqNum $= \kappa$, we define the following random variables:

- N_{b_κ}: number of retransmissions caused by bit-errors in spite of transferred packet being *in-sequence* (i.e., with seqNum equal to the next expected sequence number rcvNxt maintained by the RWP receiver).
- N_{os_κ}: number of retransmissions caused by out-of-sequence errors.

As described below, N_{b_κ} is independent of the retransmission scheme (selective retransmission, SR and go-back-N retransmission, GBR), but N_{os_κ} depends on it.

To derive N_{os_κ} for GBR, we make the following assumption:

Assumption A: The number of un-acknowledged transferred packets is always equal to W[5].

Under **Assumption A** with GBR, we have the following proposition:

Proposition 1: N_{os_κ}, which is dependent on the retransmission scheme, is given by

$$N_{os_\kappa} = \begin{cases} 0, & \text{for SR} \\ \displaystyle\sum_{i=1}^{W-1} N_{b_{\kappa-i}}, & \text{for GBR}, \end{cases} \tag{29}$$

where W is defined as the window size given in **Assumption A**.

Proof: With SR, the value of N_{os_κ} becomes zero for any $\kappa \in \mathcal{N}_0$ since no out-of-sequence error occurs.

Next, consider the case of GBR. Whenever a transferred packet with seqNum equal to rcvNxt is corrupted and discarded, transferred packets following that packet (or with seqNum greater than rcvNxt) which have been received until a retransmitted transferred packet (i.e., seqNum equal to rcvNxt) arrives at the receiver are also discarded due to out-of-sequence errors. Since the number of unacknowledged transferred packets is always equal to W from **Assumption A**, the number of such discarded packets is given by (29). ■

[5]This assumption can be justified in the following case, where (1) “cumulated” acknowledgement scheme is employed, (2) an RWP sender always has “at least one” message to be sent, and (3) window flow control with “fixed” window size is performed.

Re-arranging the sequence $\{X_{q_t}\}$, we constitute a sequence $\{\hat{X}_{q_t}\}$ expressed as

$$\{\hat{X}_{q_t}; t \in \mathcal{N}_0\} = \Big\{ \overbrace{\underbrace{X_{p_0}, \cdots, X_{p_0}}_{N_{b_0}}, \underbrace{X_{p_0}, \cdots, X_{p_0}}_{N_{os_0}}, X_{p_0}}^{N_{b_0}+N_{os_0}+1}, \cdots\cdots, \overbrace{\underbrace{X_{p_\kappa}, \cdots, X_{p_\kappa}}_{N_{b_\kappa}}, \underbrace{X_{p_\kappa}, \cdots, X_{p_\kappa}}_{N_{os_\kappa}}, X_{p_\kappa}}^{N_{b_\kappa}+N_{os_\kappa}+1}, \cdots\cdots \Big\},$$

where transferred packets with the same seqNum form a group. In APPENDIX II, we explain an example of the original size-sequence of transferred packets $\{X_{q_t}\}$ and the re-arranged size-sequence $\{\hat{X}_{q_t}\}$.

We denote the stationary distribution function of the sequence $\{\hat{X}_{q_t}\}$ by $F_{\hat{X}_q}(\cdot)$. Since the random variable X_{p_κ} appears $N_{b_\kappa} + N_{os_\kappa} + 1$ times consecutively in the size-subsequence of the transferred packets with $\mathsf{seqNum} = \kappa$ due to RPSP, we have

$$F_{\hat{X}_q}(x) = \frac{\displaystyle\int_{y=0}^{x} E[N_{b_\kappa} + N_{os_\kappa} + 1 \,|\, X_{p_\kappa} = y] dF_{X_p}(y)}{E[N_{b_\kappa} + N_{os_\kappa} + 1]}. \tag{30}$$

Note that the stationary distribution of $\hat{X}_{q_t}$ in the sequence $\{\hat{X}_{q_t}\}$ is equal to that of X_{q_t} in the sequence $\{X_{q_t}\}$, since a transferred packet is corrupted independently of the other transferred packets, owning to the Bernoulli bit-error model.

Example 3: Consider when generated packets have a common size l_c, i.e., $F_{X_p}(x) = \mathbf{1}(x - l_c)$. From (30), $F_{\hat{X}_q}(x)$ has the same form.

B. Derivation of the mean transferred packet size

Corresponding to X_{p_κ}, we introduce a random variable $Y_{p_\kappa}^{(m)}$ as[6]

$$Y_{p_\kappa}^{(m)} \triangleq (N_{b_\kappa} + N_{os_\kappa} + 1) \cdot X_{p_\kappa}^{m}, \qquad \text{for } m = 0, 1. \tag{31}$$

Then, the following proposition holds.

Proposition 2: The mean transferred packet size ℓ_q is given by

$$\ell_q \triangleq E[\hat{X}_{q_t}] = E[X_{q_t}] = \frac{E[Y_{p_\kappa}^{(1)}]}{E[Y_{p_\kappa}^{(0)}]}. \tag{32}$$

Proof: From (30), we have

$$\begin{aligned} \ell_q &= \int_0^\infty x dF_{\hat{X}_q}(x) \\ &= \frac{\displaystyle\int_{x=0}^{\infty} x \cdot E[N_{b_\kappa} + N_{os_\kappa} + 1 \,|\, X_{p_\kappa} = x] dF_{X_p}(x)}{E[N_{b_\kappa} + N_{os_\kappa} + 1]} \\ &= \frac{E[(N_{b_\kappa} + N_{os_\kappa} + 1) X_{p_\kappa}]}{E[N_{b_\kappa} + N_{os_\kappa} + 1]} \\ &= \text{Eq. (32)}. \end{aligned}$$

■

Explicit expressions of $E[Y_{p_\kappa}^{(m)}]$ for $m = 0, 1$ are given in the following proposition.

[6] The random variables $Y_{p_\kappa}^{(0)}$ and $Y_{p_\kappa}^{(1)}$ represent the total number of times and the total number of bits that an RWP-sender transmitted until an RWP-receiver has received the transferred packet correctly, respectively, for the size subsequence of transferred packets with $\mathsf{seqNum} = \kappa$.

Proposition 3: We have

$$E[Y_{p_\kappa}^{(m)}] = E[X_{p_\kappa}^m] + \Phi_b^{(m)} + \Phi_{os}^{(m)}, \tag{33}$$

$$E[X_{p_\kappa}^m] = \begin{cases} \pi_{\mathrm{E}}\ell_m, & \text{for } m = 1 \\ 1, & \text{for } m = 0, \end{cases} \tag{34}$$

$$\begin{aligned} \Phi_b^{(m)} &\triangleq E[N_{b_\kappa} \cdot X_{p_\kappa}^m] \\ &= (1 - \pi_{\mathrm{E}})\varphi(m, p_b, F_{X_{p_{\mathrm{B}}}}) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s}\varphi(m, p_b, F_{X_{p_{\mathrm{E}_s}}}), \end{aligned} \tag{35}$$

$$\begin{aligned} \Phi_{os}^{(m)} &\triangleq E[N_{os_\kappa} \cdot X_{p_\kappa}^m] \\ &= \sum_{i=1}^{W-1} \sum_{\alpha \in S_Z} \sum_{\beta \in S_Z} \varphi(0, p_b, F_{X_{p_\alpha}}) \int_0^\infty x^m dF_{X_{p_\beta}}(x) \pi_\alpha p_{Z_{\alpha\beta}}^{(i)}, \quad \text{for GBR,} \end{aligned} \tag{36}$$

where $p_{Z_{\alpha\beta}}^{(i)}$ is the (α, β)th entry of the i-step transition probability matrix $\boldsymbol{P}_Z^i$, and

$$\varphi(m, p_b, F_{X_{p_\tau}}) \triangleq \int_0^\infty \frac{p_b(x)x^m}{1 - p_b(x)} dF_{X_{p_\tau}}(x) \tag{37a}$$

$$= \int_0^\infty \frac{x^m}{1 - p_b(x)} dF_{X_{p_\tau}}(x) - \int_0^\infty x^m dF_{X_{p_\tau}}(x), \quad \tau \in S_Z. \tag{37b}$$

Here, the value of $\Phi_{os}^{(m)}$ for SR is zero.

Proof: From (31), we have

$$E[Y_{p_\kappa}^{(m)}] = E[X_{p_\kappa}^m] + E[N_{b_\kappa} \cdot X_{p_\kappa}^m] + E[N_{os_\kappa} \cdot X_{p_\kappa}^m]. \tag{38}$$

Equation (34) can be easily derived from $E[X_{p_\kappa}^1] = \ell_p$ and $E[X_{p_\kappa}^0] = E[1] = 1$.

We will prove (35) and (36) as follows.

Proof of Eq. (35)

We have

$$\Phi_b^{(m)} = E[N_{b_\kappa} \cdot X_{p_\kappa}^m] = \sum_{n=0}^{\infty} \int_{x=0}^{\infty} E[N_{b_\kappa} \cdot X_{p_\kappa}^m \,|\, N_{b_\kappa} = n, X_{p_\kappa} = x] \Pr(N_{b_\kappa} = n \,|\, X_{p_\kappa} = x) dF_{X_p}(x). \tag{39}$$

Due to RPSP, we obtain

$$E[N_{b_\kappa} \cdot X_{p_\kappa}^m \,|\, N_{b_\kappa} = n, X_{p_\kappa} = x] = n \cdot x^m. \tag{40}$$

Furthermore, the Bernoulli bit-erroneous model leads to

$$\Pr(N_{b_\kappa} = n \,|\, X_{p_\kappa} = x) = (1 - p_b(x))\, p_b(x)^n. \tag{41}$$

Substituting (40) and (41) into (39) yields

$$\begin{aligned} \Phi_b^{(m)} &= \sum_{n=0}^{\infty} \int_0^\infty n\,(1 - p_b(x))\, p_b(x)^n x^m dF_{X_p}(x) \\ &= \int_0^\infty \frac{p_b(x)x^m}{1 - p_b(x)} dF_{X_p}(x). \end{aligned} \tag{42}$$

Finally, by applying (10) and (37a) to (42), we can rewrite $\Phi_b^{(m)}$ as

$$\begin{aligned} \Phi_b^{(m)} &= (1 - \pi_{\mathrm{E}}) \int_0^\infty \frac{p_b(x)x^m}{1 - p_b(x)} dF_{X_{p_{\mathrm{B}}}}(x) + \sum_{s=1}^{\infty} \pi_{\mathrm{E}_s} \int_0^\infty \frac{p_b(x)x^m}{1 - p_b(x)} dF_{X_{p_{\mathrm{E}_s}}}(x) \\ &= \text{Eq. (35).} \end{aligned}$$

Proof of Eq. (36)

From (29), the value of $\Phi_{os}^{(m)}$ for SR is shown to be equal to zero. On the other hand, the value of $\Phi_{os}^{(m)}$ for GBR is given by

$$\Phi_{os}^{(m)} = \sum_{i=1}^{W-1} E[N_{b_{\kappa-i}} \cdot X_{p_\kappa}^m] \qquad \text{for GBR.} \tag{43}$$

In the following, we will show that

$$E[N_{b_{\kappa-i}} \cdot X_{p_\kappa}^m] = \sum_{\alpha\in S_Z}\sum_{\beta\in S_Z} \varphi(0, p_b, F_{X_{p_\alpha}}) \int_0^\infty x^m dF_{X_{p_\beta}}(x)\pi_\alpha p_{Z_{\alpha\beta}}^{(i)}, \tag{44}$$

since (36) follows immediately from (43) and (44).

From the conditional independence of $X_{p_{\kappa-i}}$ and X_{p_κ} given $Z_{\kappa-i}$ and Z_κ, we obtain

$$\begin{aligned} E[N_{b_{\kappa-i}} \cdot X_{p_\kappa}^m] &= E\{E[N_{b_{\kappa-i}} \cdot X_{p_\kappa}^m \,|\, Z_{\kappa-i}, Z_\kappa]\} \\ &= \sum_{\alpha\in S_Z}\sum_{\beta\in S_Z} \Pr(Z_{\kappa-i}=\alpha, Z_\kappa=\beta)\cdot E[N_{b_{\kappa-i}} \,|\, Z_{\kappa-i}=\alpha]E[X_{p_\kappa}^m \,|\, Z_\kappa=\beta], \end{aligned} \tag{45}$$

where $\Pr(Z_{\kappa-i}=\alpha, Z_\kappa=\beta)$, $E[N_{b_{\kappa-i}}|Z_{\kappa-i}=\alpha]$ and $E[N_{b_{\kappa-i}}|Z_{\kappa-i}=\alpha]$ are respectively given by

$$\begin{aligned} \Pr(Z_{\kappa-i}=\alpha, Z_\kappa=\beta) &= \Pr(Z_{\kappa-i}=\alpha)\Pr(Z_\kappa=\beta \,|\, Z_{\kappa-i}=\alpha) \\ &= \pi_\alpha p_{Z_{\alpha\beta}}^{(i)}, \end{aligned} \tag{46}$$

$$\begin{aligned} E[N_{b_{\kappa-i}} \,|\, Z_{\kappa-i}=\alpha] &= \sum_{n=0}^\infty \int_0^\infty n\,(1-p_b(x))\,p_b(x)^n dF_{X_{p_\alpha}}(x) \\ &= \int_0^\infty \frac{p_b(x)}{1-p_b(x)} dF_{X_{p_\alpha}}(x) \\ &= \varphi(0, p_b, F_{X_{p_\alpha}}), \end{aligned} \tag{47}$$

$$E[X_{p_\kappa}^m \,|\, Z_\kappa=\beta] = \int_0^\infty x^m dF_{X_{p_\beta}}(x). \tag{48}$$

Hence, (44) can be derived by substituting (46) - (48) into (45). ■

Example 4: Consider when $p_b(x) = c, 0 \le c < 1$, that is, the transferred packet loss probability $p_b(x)$ is independent of x and constant, equal to c. From (42), we have

$$\Phi_b^{(m)} = \int_0^\infty \frac{c\cdot x^m}{1-c} dF_{X_p}(x) = \frac{c\cdot \ell_p^m}{1-c}.$$

Furthermore, from (36) and $\varphi(0, p_b, F_{X_{p_{\mathrm{B}_\alpha}}}) = \dfrac{c}{1-c}$, we get

$$\begin{aligned} \Phi_{os}^{(m)} &= \sum_{i=1}^{W-1}\sum_{\alpha\in S_Z}\sum_{\beta\in S_Z} \frac{c}{1-c}\int_0^\infty x^m dF_{X_{p_\beta}}(x)\pi_\alpha p_{Z_{\alpha\beta}}^{(i)} \\ &= \frac{c}{1-c}\sum_{i=1}^{W-1}\sum_{\beta\in S_Z}\int_0^\infty x^m dF_{X_{p_\beta}}(x)\sum_{\alpha\in S_Z}\pi_\alpha p_{Z_{\alpha\beta}}^{(i)} \\ &= \frac{c(W-1)\ell_p^m}{1-c}, \qquad \text{for GBR,} \end{aligned}$$

by utilizing

$$\sum_{\alpha\in S_Z}\pi_\alpha p_{Z_{\alpha\beta}}^{(i)} = \sum_{\alpha\in S_Z}\Pr(Z_{\kappa-i}=\alpha, Z_\kappa=\beta) = \pi_\beta.$$

Finally, we obtain

$$\ell_q = \frac{\ell_p + \dfrac{c\,\ell_p}{1-c} + \dfrac{c\,(W-1)\ell_p}{1-c}}{1 + \dfrac{c}{1-c} + \dfrac{c\,(W-1)}{1-c}} = \ell_p. \tag{49}$$

V. Effect of RPSP

In this section, we discuss the effect of RPSP when message sizes are assumed to be exponentially distributed, since this assumption leads to concrete expressions of the mean sizes of generated and transferred packets. We use ℓ_q/ℓ_p, the ratio of the mean size of transferred packets to that of generated packets, as a measure of the effect of RPSP. We can conceive that a stronger effect of RPSP appears as ℓ_q/ℓ_p becomes larger, whereas the ratio is equal to 1 for the following network environments where RPSP has no effect (i.e., $F_{X_p}(x) = F_{X_q}(x)$ for any $x \geq 0$):

- all generated packet sizes have the same value (see *Example 3*), or
- no transferred packet loss happens, or even though transferred packet loss happens, its loss probability does not depend on the transferred packet size at all (see *Example 4*).

A. Mean size of transferred packets

For analytical tractability, we assume that message sizes are subject to an exponential distribution function $F^e_{X_m}(\cdot)$ with mean ℓ^e_m as considered in *Example 1*. From the memoryless property of the exponential distribution, we can obtain an explicit expression of the mean transferred packet size ℓ^e_q.

Proposition 4: ℓ^e_q is given by

$$\ell^e_q = \frac{E[Y^{(1)}_{p_\kappa}]}{E[Y^{(0)}_{p_\kappa}]}, \tag{50}$$

where

$$E[Y^{(m)}_{p_\kappa}] = E[X^m_{p_\kappa}] + \Phi^{(m)}_b + \Phi^{(m)}_{os}, \tag{51}$$

$$E[X^m_{p_\kappa}] \triangleq \ell^{em}_p = \begin{cases} \pi^e_{\mathrm{E}}\ell^e_m, & \text{for } m=1 \\ 1, & \text{for } m=0, \end{cases} \tag{52}$$

$$\Phi^{(m)}_b \triangleq E[N_{b_\kappa} \cdot X^m_{p_\kappa}] = (1-\pi^e_{\mathrm{E}})\varphi(m, p_b, F_{X_{p_{\mathrm{B}}}}) + \pi^e_{\mathrm{E}}\varphi(m, p_b, F^e_{X_{p_{\mathrm{E}}}}), \tag{53}$$

$$\Phi^{(m)}_{os} \triangleq E[N_{os_\kappa} \cdot X^m_{p_\kappa}] = (W-1)\cdot\Big((1-\pi^e_{\mathrm{E}})\,\varphi(0, p_b, F_{X_{p_{\mathrm{B}}}}) + \pi^e_{\mathrm{E}}\,\varphi(0, p_b, F^e_{X_{p_{\mathrm{E}}}})\Big)\cdot \ell^{em}_p, \quad \text{for GBR}, \tag{54}$$

$$\varphi(m, p_b, F_{X_{p_{\mathrm{B}}}}) = \frac{\ell^m_d}{(1-p_e)^{\ell_d+\ell_h}} - \ell^m_d, \tag{55}$$

$$\varphi(m, p_b, F^e_{X_{p_{\mathrm{E}}}}) = \begin{cases} \dfrac{1}{\ell^e_p}\left\{\dfrac{L^{e2}(1-e^{-\frac{\ell_d}{L^e}}) - \ell_d L^e e^{-\frac{\ell_d}{L^e}}}{(1-p_e)^{\ell_h}} - \pi^e_{\mathrm{E}}\ell^{2}_m + (1-\pi^e_{\mathrm{E}})\ell^e_m\ell_d\right\}, & \text{for } m=1 \\ \dfrac{1}{\ell^e_p}\left\{\dfrac{L^e(1-e^{-\frac{\ell_d}{L^e}})}{(1-p_e)^{\ell_h}} - \ell^e_p\right\}, & \text{for } m=0, \end{cases} \tag{56}$$

with $L^e \triangleq \dfrac{\ell^e_m}{\ell^e_m \log(1-p_e)+1}$.

Proof: Equations (50) and (51) are obviously derived from (32) and (33), respectively. Equation (52) can also be obviously derived from (20) and (34). Clearly, (53) can be obtained from (20) and (35). Furthermore, $\varphi(m, p_b, F_{X_{p_{\mathrm{B}}}})$ and $\varphi(m, p_b, F^e_{X_{p_{\mathrm{E}}}})$ are derived from (4) and (19), respectively.

Next, we prove (54) . From (4) and (19), $\varphi(0, p_b, F^e_{X_{p_{\mathrm{B}_r}}})$ and $\varphi(0, p_b, F^e_{X_{p_{\mathrm{E}_s}}})$ can be written as

$$\varphi(0, p_b, F^e_{X_{p_{\mathrm{B}_r}}}) = \varphi(0, p_b, F_{X_{p_{\mathrm{B}}}}), \quad \forall r \in \mathcal{N}, \tag{57}$$
$$\varphi(0, p_b, F^e_{X_{p_{\mathrm{E}_s}}}) = \varphi(0, p_b, F^e_{X_{p_{\mathrm{E}}}}), \quad \forall s \in \mathcal{N}. \tag{58}$$

Thus, from (57) and (58), we obtain

$$\Phi^{(m)}_{os} = \sum_{i=1}^{W-1} \sum_{\zeta\in\{B,E\}} \sum_{\eta\in\{B,E\}} \varphi(0, p_b, F_{X_{p_\zeta}}) \int_0^\infty x^m dF_{X_{p_\eta}}(x) \sum_{\alpha\in\zeta}\sum_{\beta\in\eta} \pi_\alpha p^{(i)}_{Z_{\alpha\beta}}. \tag{59}$$

where subsets B and E partitioning state space S_Z (i.e., $S_Z = B \cup E$) represent $B \triangleq \{\mathrm{B}_1, \mathrm{B}_2, \cdots\}$ and $E \triangleq \{\mathrm{E}_1, \mathrm{E}_2, \cdots\}$, respectively.

Definitely, the transition probability matrix $\boldsymbol{P}_Z$ of the Markov chain $\{Z_\kappa\}$ given in (3) is "lumpable" with respect to a partition of S_Z in a subset E (for the definition of "lumpability", see [16]). Hence, the stochastic process $\{\hat{Z}_\kappa\}$ with the state space $S_{\hat{Z}} \triangleq \{\hat{\mathrm{E}}, \mathrm{B}_1, \mathrm{B}_2, \cdots\}$, which is formed from $\{Z_\kappa\}$ by aggregating a subset E into a macro state $\hat{\mathrm{E}}$, can also be expressed as a Markov chain. Its dynamics can easily be derived from $\{Z_\kappa\}$ with one-step transition probability matrix $\boldsymbol{P}_{\hat{Z}} = [p_{\hat{Z}_{\alpha\beta}}, \alpha \in S_{\hat{Z}}, \beta \in S_{\hat{Z}}]$ given by

$$p_{\hat{Z}_{\alpha\beta}} = \begin{cases} 1 - u_1, & \text{for } (\alpha,\beta) = (\hat{\mathrm{E}}, \hat{\mathrm{E}}) \\ u_1, & \text{for } (\alpha,\beta) = (\hat{\mathrm{E}}, \mathrm{B}_1) \\ 1 - \dfrac{u_{k+1}}{u_k}, & \text{for } (\alpha,\beta) = (\mathrm{B}_k, \hat{\mathrm{E}}) \text{ and } \forall k \in \mathcal{N} \\ \dfrac{u_{k+1}}{u_k}, & \text{for } (\alpha,\beta) = (\mathrm{B}_k, \mathrm{B}_{k+1}) \text{ and } \forall k \in \mathcal{N} \\ 0, & \text{otherwise.} \end{cases} \tag{60}$$

In particular, if message sizes are exponentially distributed, then $\boldsymbol{P}_{\hat{Z}}$ is simplified as

$$p_{\hat{Z}_{\alpha\beta}} = \begin{cases} 1 - u^e, & \text{for } (\alpha,\beta) = (\hat{\mathrm{E}}, \hat{\mathrm{E}}) \\ u^e, & \text{for } (\alpha,\beta) = (\hat{\mathrm{E}}, \mathrm{B}_1) \\ 1 - u^e, & \text{for } (\alpha,\beta) = (\mathrm{B}_k, \hat{\mathrm{E}}) \text{ and } \forall k \in \mathcal{N} \\ u^e, & \text{for } (\alpha,\beta) = (\mathrm{B}_k, \mathrm{B}_{k+1}) \text{ and } \forall k \in \mathcal{N} \\ 0, & \text{otherwise,} \end{cases} \tag{61}$$

from (15). Letting $\pi_{\hat{\mathrm{E}}} \triangleq \Pr(\hat{Z}_\kappa = \hat{\mathrm{E}}) (= \pi^e_{\mathrm{E}})$, we obtain

$$\begin{cases} \displaystyle\sum_{\alpha\in E}\sum_{\beta\in E} \pi_\alpha p^{(i)}_{Z_{\alpha\beta}} = \pi_{\hat{\mathrm{E}}} p^{(i)}_{\hat{Z}_{\hat{\mathrm{E}}\hat{\mathrm{E}}}} = \pi^2_{\hat{\mathrm{E}}}, \\ \displaystyle\sum_{\alpha\in E}\sum_{\beta\in B} \pi_\alpha p^{(i)}_{Z_{\alpha\beta}} = \pi_{\hat{\mathrm{E}}} \sum_{\beta\in B} p^{(i)}_{\hat{Z}_{\hat{\mathrm{E}}\beta}} = \pi_{\hat{\mathrm{E}}}(1 - \pi_{\hat{\mathrm{E}}}), \\ \displaystyle\sum_{\alpha\in B}\sum_{\beta\in E} \pi_\alpha p^{(i)}_{Z_{\alpha\beta}} = \sum_{\alpha\in B} \pi_\alpha p^{(i)}_{\hat{Z}_{\alpha\hat{\mathrm{E}}}} = (1 - \pi_{\hat{\mathrm{E}}})\pi_{\hat{\mathrm{E}}}, \\ \displaystyle\sum_{\alpha\in B}\sum_{\beta\in B} \pi_\alpha p^{(i)}_{Z_{\alpha\beta}} = \sum_{\alpha\in B} \pi_\alpha (1 - p^{(i)}_{\hat{Z}_{\alpha\hat{\mathrm{E}}}}) = (1 - \pi_{\hat{\mathrm{E}}})^2 \end{cases}, \tag{62}$$

since the matrix structure given in (61) yields

$$p^{(i)}_{\hat{Z}_{\mathrm{EE}}} = \pi_{\hat{\mathrm{E}}} \qquad \forall i \in \mathcal{N},$$
$$p^{(i)}_{\hat{Z}_{\mathrm{B}_r\mathrm{E}}} = \pi_{\hat{\mathrm{E}}} \qquad \forall i \in \mathcal{N}, \forall r \in \mathcal{N}.$$

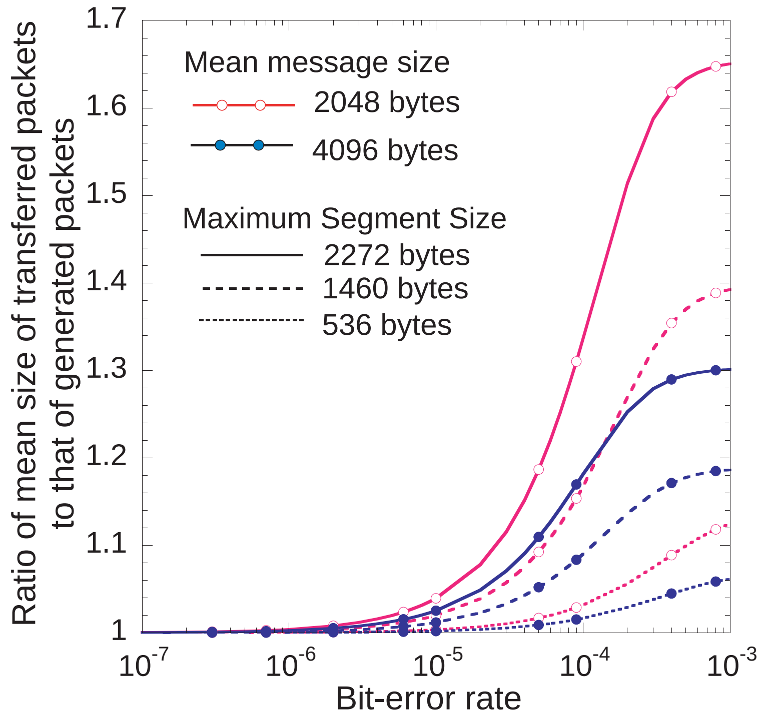

Fig. 3. Ratio of the mean size of the transferred packets to that of the generated packets ℓ_q^e/ℓ_p^e versus bit-error rate for selective retransmission scheme.

Table 1. Obtained values when $p_e = 1 \times 10^{-4}$ for selective retransmission scheme.

	Maximum Segment Size ℓ_d [bytes]		
	536	1460	2272
π_{E}^e	0.12	0.30	0.43
	0.23	0.51	0.67
ℓ_p^e [bytes]	503	1228	1744
	472	1044	1373
σ_p^e [bytes]	105	422	746
	502	1228	1744
ℓ_q^e/ℓ_p^e	1.02	1.09	1.18
	1.03	1.17	1.34

(Note: upper column; $\ell_m = 4096$ bytes, lower column; $\ell_m = 2048$ bytes.)

Consequently, from (21a) and (62), (59) can be re-written as

$$\begin{aligned}\Phi_{os}^{(m)} &= (W-1) \cdot \Big((1-\pi_{\hat{\mathrm{E}}})\, \varphi(0, p_b, F_{X_{p_{\mathrm{B}}}}) + \pi_{\hat{\mathrm{E}}}\, \varphi(0, p_b, F_{X_{p_{\mathrm{E}}}}^e) \Big) \\ &\quad \Big((1-\pi_{\hat{\mathrm{E}}}) \int_0^\infty x^m dF_{X_{p_{\mathrm{B}}}} + \pi_{\hat{\mathrm{E}}} \int_0^\infty x^m dF_{X_{p_{\mathrm{E}}}}^e \Big) \\ &= \text{Eq. (54)} \quad \text{for GBR.}\end{aligned} \tag{63}$$

■

B. *Numerical results and discussion*

In these numerical experiments, the RWP layer was assumed to be a transport (TCP) layer[7]. The control field length ℓ_h, that is TCP/IP header size, is assumed to be 40 bytes.

[7] When the RWP is assumed to be a data-link protocol such as HDLC, messages can be identified with ones generated by a network layer. However, the following discussion is valid for other cases if parameter values of message-size distribution $F_{X_m}(x)$ are changed.

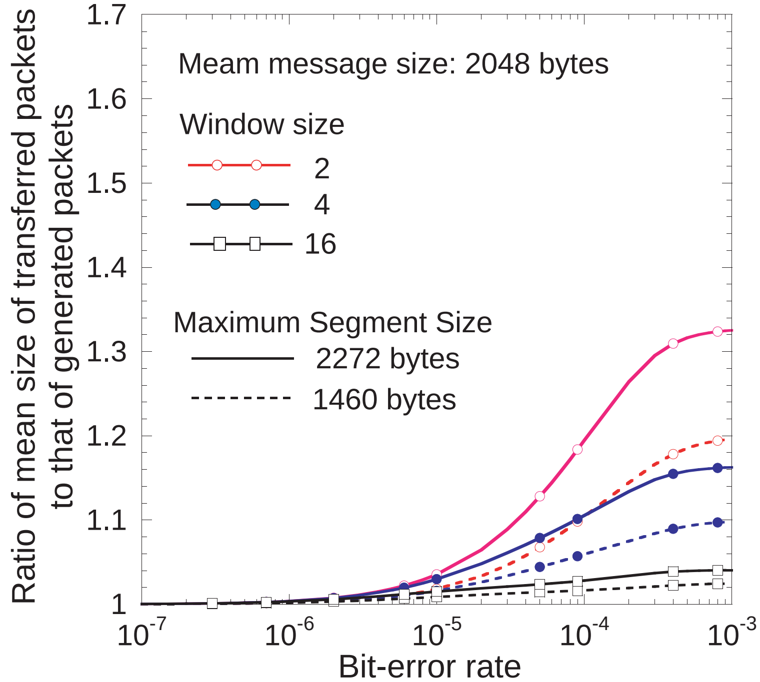

Fig. 4. Ratio of the mean size of the transferred packets to that of the generated packets ℓ_q^e/ℓ_p^e versus bit-error rate for go-back-N retransmission scheme.

Let us first investigate the effect of RPSP for selective retransmission schemes. Figure 3 shows ℓ_q^e/ℓ_p^e as a function of BER p_e for different payload sizes ℓ_d (or MSSs): 536, 1460, and 2272 bytes[8] in the case of SR. In this figure, mean message sizes of 2048 and 4096 bytes are assumed. From this figure, we find that the effect of RPSP appears when BER p_e exceeds 10^{-6}. The reason a stronger effect of RPSP appears as the BER increases is that a long transferred packet might be retransmitted more often, i.e., the realization of N_{b_κ} for a long generated packet becomes larger. In addition, large ℓ_d values such as 1460 and 2272 bytes make the effect of RPSP stronger.

To explain the reasons for this, we show the values of $\pi_{\rm E}, \ell_p^e, \sigma_p^e$, and ℓ_q^e/ℓ_p^e for different MSSs ℓ_d; 536, 1460, and 2272 bytes when p_e is 1×10^{-4}, which is the mean BER of a wireless link in an industrial environment [17], in the case of SR, as listed in Table 1. Table 1 shows the edge-packet occurrence probability $\pi_{\rm E}^e$ is relatively large (i.e., the message-segmentation occurrence probability $1-\pi_{\rm E}^e$ is relatively small), which causes a large variance in the generated packet size distribution (see *Example 2*). In particular, the effect of RPSP is significant for networks that suffer from a high BER p_e. On the other hand, when the MSS ℓ_d of 512 bytes is used, the value of ℓ_q^e/ℓ_p^e is relatively small since the body-packets are dominant. In this case (i.e., when $\pi_{\rm E}^e$ is enough small), the effect of RPSP almost disappears.

Next, we discuss the effect of RPSP in the case of go-back-N retransmission scheme. Figure 4 shows ℓ_q^e/ℓ_p^e as a function of BER p_e for different payload sizes ℓ_d of 1460 and 2272 bytes and for window sizes W of 2, 4 and 16 in the case of GBR. In this figure, mean message size is used 2048 bytes. From this figure, we find that RPSP is not negligible in the case of GBR with small window sizes if BER is high. We note that ℓ_q^e of GBR with $W=1$ is equal to that of SR (see (54)). On the other hand, for GBR with large W, ℓ_q^e/ℓ_p^e is rather small (near to 1). An intuitive explanation for this fact is that, when W is large, the number of retransmissions of corrupted transferred packets increases but the transferred packet retransmission probability due to out-of-sequence errors caused by GBR becomes independent of the corruption probability of the "own" transferred packets (i.e., the probability of retransmission of short transferred packets as well as long transferred packets increases).

Proposition 5: Letting $\ell_{q_{max}}$ be a finite limit of the mean transferred packet size as $p_e \to 1$, we obtain:

$$\ell_{q_{max}} = \begin{cases} \ell_d, & \text{for SR} \\ \dfrac{\ell_d + (W-1)\ell_p^e}{W}, & \text{for GBR.} \end{cases} \tag{64}$$

[8]The MSS values of 536, 1460, and 2272 bytes are the common default values for hosts when the path MTU (maximum transmission unit) discovery option is not used, Ethernet MTU (that is 1500) - ℓ_h, and IEEE 802.11b MTU (that is 2312) - ℓ_h, respectively.

Proof: From (55) and (56), we have:

$$\lim_{p_e\to 1}\Phi_b^{(1)} \approx \frac{(1-\pi_{\mathrm{E}}^e)\ell_d}{(1-p_e)^{\ell_d+\ell_h}} - \frac{\pi_{\mathrm{E}}^e(L^{e2}e^{-\frac{\ell_d}{\ell_m^e}}+\ell_d L^e e^{-\frac{\ell_d}{\ell_m^e}})}{\ell_p^e(1-p_e)^{\ell_d+\ell_h}}, \tag{65}$$

$$\lim_{p_e\to 1}\Phi_b^{(0)} \approx \frac{1-\pi_{\mathrm{E}}^e}{(1-p_e)^{\ell_d+\ell_h}} - \frac{\pi_{\mathrm{E}}^e L^e e^{-\frac{\ell_d}{\ell_m^e}}}{\ell_p^e(1-p_e)^{\ell_d+\ell_h}}. \tag{66}$$

From $\lim_{p_e\to 1}\Phi_b^{(1)} \gg \ell_p^e$ and $\lim_{p_e\to 1}\Phi_b^{(0)} \gg 1$, $\ell_{q_{max}}$ for SR can be written as:

$$\begin{aligned}
\ell_{q_{max}} &= \lim_{p_e\to 1}\frac{\ell_p^e+\Phi_b^{(1)}}{1+\Phi_b^{(0)}} = \frac{\lim\limits_{p_e\to 1}\Phi_b^{(1)}}{\lim\limits_{p_e\to 1}\Phi_b^{(0)}} \\
&\approx \frac{(1-\pi_{\mathrm{E}}^e)\ell_d - \frac{\pi_{\mathrm{E}}^e}{\ell_p^e}(L^{e2}e^{-\frac{\ell_d}{\ell_m^e}}+\ell_d L^e e^{-\frac{\ell_d}{\ell_m^e}})}{(1-\pi_{\mathrm{E}}^e) - \frac{\pi_{\mathrm{E}}^e}{\ell_p^e}\pi_{\mathrm{E}}^e L^e e^{-\frac{\ell_d}{\ell_m^e}}} \\
&= \ell_d, \quad (L^e\to 0) \qquad \text{for SR.}
\end{aligned} \tag{67}$$

Similarly, $\ell_{q_{max}}$ for GBR can be expressed in the following:

$$\begin{aligned}
\ell_{q_{max}} &= \lim_{p_e\to 1}\frac{\ell_p^e+\Phi_b^{(1)}+\Phi_{os}^{(1)}}{1+\Phi_b^{(0)}+\Phi_{os}^{(0)}} = \frac{\lim\limits_{p_e\to 1}\Phi_b^{(1)}+\lim\limits_{p_e\to 1}\Phi_{os}^{(1)}}{\lim\limits_{p_e\to 1}\Phi_b^{(0)}+\lim\limits_{p_e\to 1}\Phi_{os}^{(0)}} \\
&\approx \frac{(1-\pi_{\mathrm{E}}^e)\ell_d - \frac{\pi_{\mathrm{E}}^e}{\ell_p^e}(L^{e2}e^{-\frac{\ell_d}{\ell_m^e}}+\ell_d L^e e^{-\frac{\ell_d}{\ell_m^e}}) + \ell_p^e(W-1)\left\{(1-\pi_{\mathrm{E}}^e)-\frac{\pi_{\mathrm{E}}^e}{\ell_p^e}L^e e^{-\frac{\ell_d}{\ell_m^e}}\right\}}{(1-\pi_{\mathrm{E}}^e) - \frac{\pi_{\mathrm{E}}^e}{\ell_p^e}\pi_{\mathrm{E}}^e L^e e^{-\frac{\ell_d}{\ell_m^e}} + (W-1)\left\{(1-\pi_{\mathrm{E}}^e)-\frac{\pi_{\mathrm{E}}^e}{\ell_p^e}L^e e^{-\frac{\ell_d}{\ell_m^e}}\right\}}, \\
&= \frac{(1-\pi_{\mathrm{E}}^e)\ell_d + (1-\pi_{\mathrm{E}}^e)(W-1)\ell_p^e}{(1-\pi_{\mathrm{E}}^e)+(1-\pi_{\mathrm{E}}^e)(W-1)} \quad (L^e\to 0) \\
&= \frac{\ell_d+(W-1)\ell_p^e}{W} \qquad \text{for GBR,}
\end{aligned} \tag{68}$$

from

$$\lim_{p_e\to 1}\Phi_{os}^{(1)} \approx \ell_p^e(W-1)\left\{\frac{1-\pi_{\mathrm{E}}^e}{(1-p_e)^{\ell_d+\ell_h}} - \frac{\pi_{\mathrm{E}}^e L^e e^{-\frac{\ell_d}{\ell_m^e}}}{\ell_p^e(1-p_e)^{\ell_d+\ell_h}}\right\}, \tag{69}$$

$$\lim_{p_e\to 1}\Phi_{os}^{(0)} \approx (W-1)\left\{\frac{1-\pi_{\mathrm{E}}^e}{(1-p_e)^{\ell_d+\ell_h}} - \frac{\pi_{\mathrm{E}}^e L^e e^{-\frac{\ell_d}{\ell_m^e}}}{\ell_p^e(1-p_e)^{\ell_d+\ell_h}}\right\}. \tag{70}$$

Therefore, (64) can be derived. ■

Remark 2: Proposition 5 implies that $\ell_{q_{max}}$ for GBR depends on window size W and it converges to ℓ_p^e as $W\to\infty$, as stated before.

The above observations indicate that performance models for versions of Reno/NewReno/SACK of TCP, which neglect RPSP as described in [10], might lead to overestimations for the following reasons (the detailed discussions can be found in [**?**]).

- Error recovery of the Reno/NewReno/SACK versions of TCP is based on the SR-scheme.

- Message sizes of HTTP-traffic, which has become one of the largest consumers of Internet resources [18], are randomly distributed [12][9],
- The mean transferred packet size ℓ_q influences TCP performance implicitly, because the factors affecting TCP performance include the following:
  - round-trip time that contains the transmission delay proportional to ℓ_q because the delay is given by the size of the frame (containing the transferred packet) divided by the link-speed,
  - transferred packet corruption probability for an SR-scheme, since it is given by $E[p_b(X_{q_t})] = \int_0^\infty p_b(X_{q_\kappa}) dF_{\hat{X}_q}(x) = \frac{E[N_{b_\kappa}]}{E[N_{b_\kappa}+1]}$ with $E[N_{b_\kappa}]$ being a function of the size-distribution of the transferred packet.

VI. Conclusions and future work

In this paper, we discussed the retransmitted packet size preservation (RPSP) property that all transferred packets at retransmissions have the same size as that at the original transmission, i.e., identical to the packet generated from a message. To analyze the effect of RPSP in an environment where message segmentation occurs, we presented a Markov model of size-sequences of the generated and transferred packets, for which the message-segmentation and the error-recovery functions with the RPSP-property are taken into account, respectively. Using this model, we derived the size-distributions of the generated and transferred packets. Furthermore, we obtained analytical expressions of the respective mean size. In addition, we analyzed the effect of RPSP by investigating the ratio of the mean transferred packet size to the mean generated packet size, i.e., the extent of the impact of RPSP. From numerical results when the message-sizes are exponentially distributed, we demonstrated that the effect of RPSP cannot be negligible for networks that suffer from a high BER (e.g., 1×10^{-4}, which is the mean BER of a wireless link in an industrial environment), when the message-segmentation occurrence probability is relatively small, such as payload sizes of 1460 and 2272 bytes for mean message size of 2048 bytes, and selective repeat retransmission or go-back-N retransmission with small window sizes is performed.

The remaining issues include investigating the effect of RPSP in network environments with a *burst* bit-error model and with an *empirical* message-size model.

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[9] According to HTTP-traffic measurements including [12], the size of messages, that is Web-objects, has a Pareto or log-normal distribution rather than an exponential distribution. However, it is difficult to derive strict expressions of $F_{X_p}(\cdot)$ and $F_{X_q}(\cdot)$ for such suitable distributions. The many topics for future work include deriving good approximate expressions for them, as described in Section VI.

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APPENDIX I
DERIVATION OF EQUATION (3): $p_{Z_{\alpha\beta}}$ ONE-STEP TRANSITION PROBABILITIES OF SEQUENCE $\{Z_\kappa\}$

This appendix gives the derivation of (3). Here, we assume that the κth generated packet contains the ith message of size X_{m_i}, and we calculate the one-step transition probabilities from Z_κ of $\mathrm{E}_s, \forall s \in \mathcal{N}$, or $\mathrm{B}_r, \forall r \in \mathcal{N}$, respectively, to any state $Z_{\kappa+1}$: $p^{(1)}_{Z_{\alpha\beta}}, \alpha \in S_z, \beta \in S_Z$.

- **When $Z_\kappa = \mathrm{E}_s, \forall s \in \mathcal{N}$ (i.e., when the jth generated packet is an edge-packet)**
  In this case, the $(\kappa+1)$th packet is the first generated packet from the $(i+1)$th message. There are only two cases:
  - **if the size of the $(i+1)$th message is less than or equal to ℓ_d**
    Since the $(i+1)$th message is not segmented, the $(\kappa+1)$ generated packet is identical to it. Then, from the *i.i.d.* message size assumption,

$$\begin{aligned}
Z_{\kappa+1} &= \mathrm{E}_1, \\
p_{Z_{\mathrm{E}_s\mathrm{E}_1}} &\stackrel{\triangle}{=} \Pr(Z_{\kappa+1} = \mathrm{E}_1 | Z_\kappa = \mathrm{E}_s) \\
&= \Pr(X_{m_{i+1}} \le \ell_d | (s-1)\ell_d < X_{m_i} \le s\ell_d) \\
&= \frac{\Pr((s-1)\ell_d < X_{m_i} \le s\ell_d, X_{m_{i+1}} \le \ell_d)}{\Pr((s-1)\ell_d < X_{m_i} \le s\ell_d)} \\
&= \Pr(X_{m_{i+1}} \le \ell_d) \\
&= F_{X_m}(\ell_d) = 1 - u_1.
\end{aligned}$$

  - **if the size of the $(i+1)$th message is greater than ℓ_d**

    Since the $(i+1)$th message is segmented, the $(\kappa+1)$th generated packet becomes the head body-packet. Hence, the *i.i.d.* message size assumption yields

$$\begin{aligned}
Z_{\kappa+1} &= \mathrm{B}_1, \\
p_{Z_{\mathrm{E}_s\mathrm{B}_1}} &\stackrel{\triangle}{=} \Pr(Z_{\kappa+1} = \mathrm{B}_1 | Z_\kappa = \mathrm{E}_s) \\
&= \Pr(X_{m_{i+1}} > \ell_d) \\
&= 1 - F_{X_m}(\ell_d) = u_1.
\end{aligned}$$

- **When $Z_\kappa = \mathrm{B}_r, \forall r \in \mathcal{N}$ (i.e., when the κth generated packet is a body-packet following r body-packets)**

  In this case, the $(\kappa+1)$th generated packet is the $(r+1)$th generated packet from the ith message. Again, there are only two cases:
  - **if the size of the remaining part of the ith message segmented is less than or equal to ℓ_d**

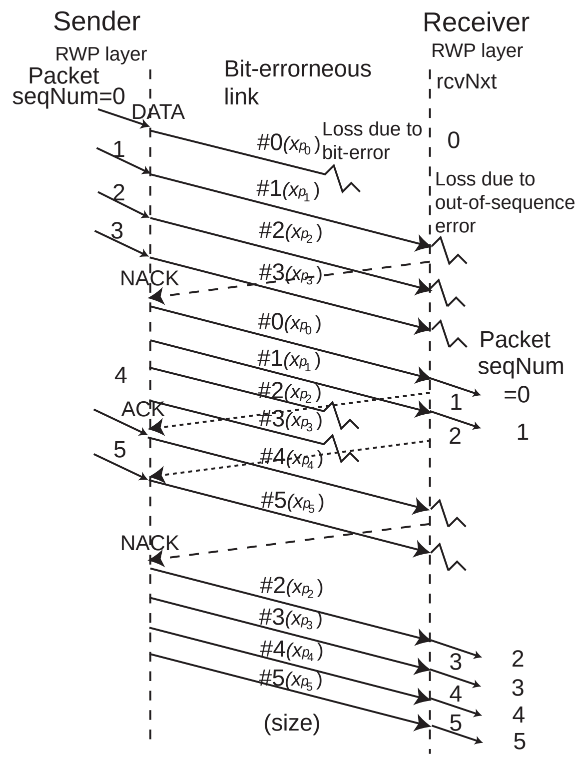

Fig. 5. Example of the transferred packet size sequence when GBR ARQ with a window size of 4 is performed.
(Note DATA: data packet, ACK: acknowledgement packet, NACK: negative acknowledgement packet)

The $(\kappa+1)$th generated packet becomes the edge-packet made of the ith message. Then,

$$
\begin{aligned}
Z_{\kappa+1} &= \mathrm{E}_{r+1}, \\
p_{Z_{\mathrm{B}_r \mathrm{E}_{r+1}}} &\stackrel{\triangle}{=} \Pr(Z_{\kappa+1} = \mathrm{E}_{r+1} | Z_\kappa = \mathrm{B}_r) \\
&= \Pr(r\ell_d < X_{m_i} \le (r+1)\ell_d | X_{m_i} > r\ell_d) \\
&= \frac{F_{X_m}((r+1)\ell_d) - F_{X_m}(r\ell_d)}{1 - F_{X_m}(r\ell_d)} = 1 - \frac{u_{r+1}}{u_r}.
\end{aligned}
$$

– **if the size of the remaining part of the ith message segmented is greater than ℓ_d**
The $(\kappa+1)$th generated packet becomes a consecutive body-packet. Therefore, we obtain:

$$
\begin{aligned}
Z_{\kappa+1} &= \mathrm{B}_{r+1}, \\
p_{Z_{\mathrm{B}_r \mathrm{B}_{r+1}}} &\stackrel{\triangle}{=} \Pr(Z_{\kappa+1} = \mathrm{B}_{r+1} | Z_\kappa = \mathrm{B}_r) \\
&= \Pr((r+1)\ell_d < X_{m_i} | X_{m_i} > r\ell_d) \\
&= \frac{1 - F_{X_m}((r+1)\ell_d)}{1 - F_{X_m}(r\ell_d)} = \frac{u_{r+1}}{u_r}.
\end{aligned}
$$

Note that transitions to any other states do not happen. Therefore, the above discussion leads to (3).

APPENDIX II
EXAMPLE OF THE ORIGINAL TRANSFERRED PACKET SIZE SEQUENCE $\{X_{q_t}\}$ AND THE RE-ARRANGED SEQUENCE $\{\hat{X}_{q_t}\}$

This Appendix gives an example of the original size-sequence of the transferred packets $\{X_{q_t}\}$ and the re-arranged size-sequence $\{\hat{X}_{q_t}\}$. Figure 5 illustrates the sequence of the transferred packets when GBR ARQ (automatic request repeat) with $W = 4$ is performed. In this figure, x_{p_κ} for $\kappa = 0, \cdots, 5$ represents the realization of the generated packet size X_{p_κ}. In this example, we assume that the first transmitted transferred packet of seqNum $= 0$, the second transmitted transferred packet of seqNum $= 2$, and the second transmitted transferred packet of seqNum $= 3$ are lost due to bit-errors (i.e., corrupted). Then, the realization of the size-sequence of transferred packets $\{X_{q_t}; t = 0, \cdots, 13\}$ is given by

$$\{X_{q_t}; t = 0, \cdots, 13\} = \{\overbrace{\dot{x}_{p_0}, \ddot{x}_{p_1}, \ddot{x}_{p_2}, \ddot{x}_{p_3}}^{W}, x_{p_0}, x_{p_1}, \overbrace{\dot{x}_{p_2}, \ddot{x}_{p_3}, \ddot{x}_{p_4}, \ddot{x}_{p_5}}^{W}, x_{p_2}, x_{p_3}, x_{p_4}, x_{p_5}\}, \tag{71}$$

where $\dot{x}_{p_\kappa}$ and $\ddot{x}_{p_\kappa}$ represent the sizes of generated packets seqNum$=\kappa$ that have suffered from bit-errors and being out of sequence, respectively. Thus, the numbers of corruptions experienced for in-sequence transferred packets N_{b_κ} for $\kappa = 0, \cdots, 5$ are given by

$$\{N_{b_\kappa}; \kappa = 0, \cdots, 5\} = \{1, 0, 1, 0, 0, 0\}. \tag{72}$$

From this illustration, the numbers of out-of-sequence errors experienced for transferred packets N_{os_κ} for $\kappa = 0, \cdots, 5$ can be expressed as

$$\{N_{os_\kappa}; \kappa = 0, \cdots, 5\} = \{0, 1, 1, 2, 1, 1\}. \tag{73}$$

We note that the retransmission of the second transmitted transferred packet of seqNum $= 3$ increments N_{os_3} rather than N_{b_3}, since it is retransmitted because of the out-of-sequence-error even if it has been corrupted. It is easy to find that (73) satisfies (29). Hence, the re-arranged transferred packet size sequence $\{\hat{X}_{q_t}; t = 0, \cdots, 13\}$ can be represented as

$$\{\hat{X}_{q_t}; t = 0, \cdots, 13\} = \{ \overbrace{\underbrace{\dot{x}_{p_0}}_{N_{b_0}}, x_{p_0}}^{N_{b_0}+N_{os_0}+1}, \overbrace{\underbrace{\ddot{x}_{p_1}}_{N_{os_1}}, x_{p_1}}^{N_{b_1}+N_{os_1}+1}, \overbrace{\underbrace{\ddot{x}_{p_2}}_{N_{os_2}}, \underbrace{\dot{x}_{p_2}}_{N_{b_2}}, x_{p_2}}^{N_{b_2}+N_{os_2}+1}, \overbrace{\underbrace{\ddot{x}_{p_3}, \ddot{x}_{p_3}}_{N_{os_3}}, x_{p_3}}^{N_{b_3}+N_{os_3}+1}, \overbrace{\underbrace{\ddot{x}_{p_4}}_{N_{os_4}}, x_{p_4}}^{N_{b_4}+N_{os_4}+1}, \overbrace{\underbrace{\ddot{x}_{p_5}}_{N_{os_5}}, x_{p_5}}^{N_{b_5}+N_{os_5}+1} \}.$$