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Covariance Analysis for Packet Size Sequence with Message Segmentations

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Abstract

In many cases, each network imposes a maximum permitted packet size called payload size. Hence, some protocols such as TCP specify a message segmentation function allows a sender to divide a single message greater than the payload size into multiple packets. In this report, we derive an analytical form of a covariance of a packet size sequence for an environment where message segmentations happen. We show that when message sizes are exponentially distributed, the packet sizes are uncorrelated, that is, the covariance is always zero because of the memoryless property of an exponential distribution, even though the message segmentations happen. However, from numerical results where HTTP messages are lognormally distributed according to an actual traffic measurement, we demonstrate that TCP-packet sizes exhibit heavily-correlated property in cases of payload sizes used commonly for TCP.

Key words – Covariance, packet sizes, message segmentation, TCP, HTTP, correlation.

I. Introduction

For queueing systems, correlated statistical property of interarrival times and/or service times is well-known to affect performance of queueing systems significantly. For example, paper [1] demonstrates that interarrival times with significant correlations across large time scale yield very large mean queueing delays compared with those obtained from a Markovian model. Hence, careful statistical analysis to investigate the correlated property is required.

Round-trip time (or one-way delay) and packet loss rate are fundamental QoS (quality of service) parameters for users of packet-based transfer networks such as the Internet [2, pp. 4–6]. They are dependent on packet sizes. In communication networks with low bit-rate links and/or high bit-error prone links such as wireless access links, the characteristics of packet sizes affect such QoS parameters significantly. To assess the QoS parameters in such communication network environments with high accuracy, therefore, we need to investigate correlation of packet size sequences, of which typical measure is well-known to be a covariance.

In many cases, each network imposes a maximum permitted packet size called payload size due to link structure (e.g., the width of a transmission slot) compliance with standard protocol specifications [3, p. 406]. Messages, that is, data units generated by applications, are frequently greater than the payload size. To convey such messages over the network, some communication protocols (such as TCP/IP [4] and RLC [5]) specify a message segmentation/reassembly function. The message segmentation function allows a sender to divide a single message greater than the payload size into multiple packets. This function sometimes yields the correlation among created packets in size. In this report, we attempt to derive an analytical form of covariance for packet size sequences with message segmentations, and investigate the effect of payload size on the covariance.

Paper [6] investigated the correlation of IP-level packet sizes for wide area networks (WANs) based on actual traffic measurements. The paper showed that the size-correlation of packets of which sessions are statistically multiplexed into the WAN diminishes, although packet sizes are often highly correlated for each session when packets are generated by applications. Major difference between previous work and our work is that our approach is theoretical, which allows us to investigate effect of any payload size on the correlation of packets sizes with a given message size distribution.

The rest of the report is organized as follows. In Section II, a model of size sequence of packets is provided, given a message size distribution and a payload size. To analyze the packet size sequence through a framework of Markov chains (MCs), we introduce an associated MC representing the packet types of two kinds (called a body packet and an edge packet), referred to as a packet-type MC. Furthermore, we develop the aggregated MC based on the packet-type MC, which allows us to calculate the packet size covariance efficiently. Section III explains their MCs. The packet size covariance can be expressed in terms of elements of an n-step transition probability matrix of the packet-type MC (or that of the aggregated packet-type MC). Section IV calculates the n-step transition-probability matrix for the aggregated packet-type MC. Then, Section V calculates the n-step transition-probability matrix for the aggregated packet-type MC based on the results obtained in Section IV. In Section VI, we derive the covariance of packet sizes using the results obtained in Sections III to V. Section VII investigates the effect of the payload size on the correlation of the TCP-packet sizes when HTTP-messages sizes are subject to the measured distribution reported in [7]. Finally, Section VIII summarizes the report.

II. Packet size sequence model[8]

Letting $X_i^{(m)} > 0$ denote the ith message size, we assume $\{X_i^{(m)}; i \in \mathcal{N}_0\}$ where $\mathcal{N}_0 \triangleq \{0, 1, 2, \ldots\}$ is a sequence of mutually independent and identically distributed (*i.i.d.*) random variables with a common distribution function $F^{(m)}(\cdot)$ of mean message-size $\ell^{(m)}$. If $X_i^{(m)}$ is greater than a payload size ℓ_d, then the ith message is divided into multiple packets with a size-sequence $\{X_{ij}^{(p)} : j = 1, \cdots, k_i\}$, where

$$\begin{cases} k_i &= \left\lceil \dfrac{X_i^{(m)}}{\ell_d} \right\rceil, \qquad\qquad \text{for } i \in \mathcal{N}_0, \\ \\ X_{ij}^{(p)} &= \begin{cases} \ell_d, & \text{for } j = 1, 2, \ldots, k_i - 1 \\ X_i^{(m)} - (k_i - 1)\ell_d, & \text{for } j = k_i. \end{cases} \end{cases} \tag{1}$$

Here operator $\lceil a \rceil$ represents the smallest integer that is greater than or equal to a. For the ith message, we refer to the j $(\leq k_i - 1)$ th packet as a "body" packet and the last (i.e., k_i th) packet as an "edge" packet. If $X_i^{(m)} \leq \ell_d$, the ith message is not segmented, and a single packet, which is identical to the original message, is generated. We also refer to this as an "edge" packet, because it satisfies definition (1).

III. Packet-type Markov chain

We constitute a stochastic process $\{X_\kappa^{(p)}; \kappa \in \mathcal{N}_0\}$, replacing the pair of epoch labels ij by an in-sequence number $\kappa \in \mathcal{N}_0$ for $\{X_{ij}^{(p)}\}$. To analyze the behavior of $\{X_\kappa^{(p)}\}$ through the framework of Markov chains, we introduce an auxiliary random variable Z_κ associated with

$X_\kappa^{(p)}$. The random variable Z_κ is defined on the state space $S^{(Z)} = \{\mathrm{D}_1, \mathrm{D}_2, \cdots ; \mathrm{E}_1, \mathrm{E}_2, \cdots\}$, where, for a message under consideration,

state $\mathbf{D}_r$ indicates that a packet is the rth body packet, and
state $\mathbf{E}_s$ indicates that a packet is an edge packet following $(s-1)$ body-packets.

The stochastic process $\{Z_\kappa\}$ can be represented as an MC (referred to as "packet-type Markov chain"), described in the following proposition:

Proposition 1: The stochastic process $\{Z_\kappa\}$ can be represented as a Markov chain having the following one-step transition-probability matrix $\boldsymbol{P}^{(Z)} = [p_{\alpha,\beta}^{(Z)}, \alpha \in S^{(Z)}, \beta \in S^{(Z)}]$ with elements

$$p_{\alpha,\beta}^{(Z)} = \begin{cases} 1 - \dfrac{u_{r+1}}{u_r}, & \text{for } (\alpha, \beta) = (\mathrm{D}_r, \mathrm{E}_{r+1}),\ r \in \mathcal{N} \\ \dfrac{u_{r+1}}{u_r}, & \text{for } (\alpha, \beta) = (\mathrm{D}_r, \mathrm{D}_{r+1}),\ r \in \mathcal{N} \\ 1 - u_1, & \text{for } (\alpha, \beta) = (\mathrm{E}_s, \mathrm{E}_1),\ s \in \mathcal{N} \\ u_1, & \text{for } (\alpha, \beta) = (\mathrm{E}_s, \mathrm{D}_1),\ s \in \mathcal{N} \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $\mathcal{N} \stackrel{\triangle}{=} \{1, 2, \cdots\}$ and

$$u_r \stackrel{\triangle}{=} \int_{r\ell_d}^{\infty} dF^{(m)}(x) = 1 - F^{(m)}(r\ell_d), \quad \text{for } r \in \mathcal{N}. \tag{3}$$

Proof: See [8, APPENDIX I]. ∎

Note that

$$u_0 = \int_0^{\infty} dF^{(m)}(x) = 1. \tag{4}$$

From the matrix structure shown in (2), it is easy to see that the transition-probability matrix $\boldsymbol{P}^{(Z)}$ is "lumpable" with respect to a partition $\{\mathcal{E}, \mathrm{D}_1, \mathrm{D}_2, \cdots,\}$ where $\mathcal{E} \stackrel{\triangle}{=} \{\mathrm{E}_1, \mathrm{E}_2, \cdots\}$, because real-valued constants $r_{\mathcal{E}\,\beta}$ exist such that the following condition holds for $\beta \in \mathcal{D} \stackrel{\triangle}{=} \{\mathrm{D}_1, \mathrm{D}_2, \cdots\}$,

$$p_{\alpha,\beta}^{(Z)} = r_{\mathcal{E}\,\beta} \stackrel{\triangle}{=} \begin{cases} u_1, & \text{for } \beta = \mathrm{D}_1, \\ 0, & \text{for } \beta \in \mathcal{D} - \{\mathrm{D}_1\}, \end{cases} \qquad \text{for } \alpha \in \mathcal{E}. \tag{5}$$

For the definition of "lumpability", see [9]. Hence, the stochastic process $\{\hat{Z}_\kappa\}$ with the state space $S^{(\hat{Z})} \stackrel{\triangle}{=} \{\hat{\mathrm{E}}, \mathrm{D}_1, \mathrm{D}_2, \cdots\}$, which is formed from $\{Z_\kappa\}$ by aggregating the subset $\mathcal{E}$ into a macro state $\hat{\mathrm{E}}$, can also be expressed as an MC (called an aggregated packet-type Markov chain). As will be described in Section V, the behavior of $\{Z_\kappa\}$ can easily be derived from $\{\hat{Z}_\kappa\}$ with a one-step transition-probability matrix $\boldsymbol{P}^{(\hat{Z})} = [p_{\alpha,\beta}^{(\hat{Z})}, \alpha \in S^{(\hat{Z})}, \beta \in S^{(\hat{Z})}]$ given by

$$p_{\alpha,\beta}^{(\hat{Z})} = \begin{cases} 1 - u_1, & \text{for } (\alpha, \beta) = (\hat{\mathrm{E}}, \hat{\mathrm{E}}), \\ 1 - \dfrac{u_{k+1}}{u_k}, & \text{for } (\alpha, \beta) = (\mathrm{D}_k, \hat{\mathrm{E}}),\ k \in \mathcal{N}, \\ \dfrac{u_{k+1}}{u_k}, & \text{for } (\alpha, \beta) = (\mathrm{D}_k, \mathrm{D}_{k+1}),\ k \in \mathcal{N}, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

IV. CALCULATION OF n-STEP TRANSITION PROBABILITY MATRIX FOR AGGREGATED MARKOV CHAINS $\{\hat{Z}_\kappa\}$[10]

We denote an n-step transition-probability matrix for the aggregated packet-type MC $\{\hat{Z}_\kappa\}$ by $\{\boldsymbol{P}^{(\hat{Z})}\}^n$. To calculate of the elements of the matrix $\{\boldsymbol{P}^{(\hat{Z})}\}^n$ effectively, firstly we introduce hitting probabilities (or first-entrance probabilities) and taboo probabilities of the aggregated packet-type MC $\{\hat{Z}_\kappa\}$.[1] Secondly, we calculate the elements of the matrix $\{\boldsymbol{P}^{(\hat{Z})}\}^n$, using the hitting and taboo probabilities.

A. Preliminary

a) Calculation of hitting probabilities: Let the hitting probability $\hat{h}_{\alpha,\beta}$ be a probability that the first visit to state β occurs at epoch n, given $\hat{Z}_0 = \alpha$. Thus,

$$\hat{h}_{\alpha,\beta}(n) \stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \beta, \hat{Z}_2 \neq \beta, \cdots, \hat{Z}_{n-1} \neq \beta, \hat{Z}_n = \beta \,|\, \hat{Z}_0 = \alpha\right),$$
$$\text{for } \alpha \in S^{(\hat{Z})},\ \beta \in S^{(\hat{Z})}, \text{ and } n \in \mathcal{N}. \tag{7}$$

We focus on the hitting probabilities $\hat{h}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n)$ and $\hat{h}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$, which are available for the calculation of a covariance. From the structure of the matrix $\boldsymbol{P}^{(Z)}$ given in (2), they are simply expressed as:

$$\begin{aligned}
\hat{h}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n) &\stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \hat{\mathrm{E}}, \hat{Z}_2 \neq \hat{\mathrm{E}}, \cdots, \hat{Z}_{n-1} \neq \hat{\mathrm{E}}, \hat{Z}_n = \hat{\mathrm{E}} \,|\, \hat{Z}_0 = \mathrm{D}_r\right)\\
&= \Pr\left(\hat{Z}_1 = \mathrm{D}_{r+1}, \hat{Z}_2 = \mathrm{D}_{r+2}, \cdots, \hat{Z}_{n-1} = \mathrm{D}_{r+n-1}, \hat{Z}_n = \hat{\mathrm{E}} \,|\, \hat{Z}_0 = \mathrm{D}_r\right)\\
&= \frac{u_{r+1}}{u_r}\frac{u_{r+2}}{u_{r+1}}\frac{u_{r+3}}{u_{r+2}}\cdots\frac{u_{r+n-1}}{u_{r+n-2}}\left(1 - \frac{u_{r+n}}{u_{r+n-1}}\right) \qquad \text{(from (2))}\\
&= \frac{u_{r+n-1} - u_{r+n}}{u_r}, \qquad \text{for } r \in \mathcal{N} \text{ and } n \in \mathcal{N},
\end{aligned} \tag{8}$$

$$\begin{aligned}
\hat{h}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n) &\stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \hat{\mathrm{E}}, \hat{Z}_2 \neq \hat{\mathrm{E}}, \cdots, \hat{Z}_{n-1} \neq \mathrm{E}, \hat{Z}_n = \hat{\mathrm{E}} \,|\, \hat{Z}_0 = \hat{\mathrm{E}}\right)\\
&= \Pr\left(\hat{Z}_1 = \mathrm{D}_{r+1}, \hat{Z}_2 = \mathrm{D}_{r+2}, \cdots, \hat{Z}_{n-1} = \mathrm{D}_{r+n-1}, \hat{Z}_n = \hat{\mathrm{E}} \,|\, \hat{Z}_0 = \mathrm{D}_r\right)\\
&= u_1 \frac{u_2}{u_1}\frac{u_3}{u_2}\cdots\frac{u_{n-1}}{u_{n-2}}\left(1 - \frac{u_n}{u_{n-1}}\right) \qquad \text{(from (2))}\\
&= u_{n-1} - u_n, \qquad \text{for } n \in \mathcal{N}.
\end{aligned} \tag{9}$$

b) Calculation of taboo probabilities: Given that the initial state is α, i.e., $\hat{Z}_0 = \alpha$, let the taboo probability $\hat{t}_{\alpha,\beta}$ be a probability that the visit to state β occurs at epoch n without returning to the initial state α. That is

$$\hat{t}_{\alpha,\beta}(n) \stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \beta, \hat{Z}_2 \neq \beta, \cdots, \hat{Z}_{n-1} \neq \ \beta, \hat{Z}_n = \beta \,|\, \hat{Z}_0 = \alpha\right),$$
$$\text{for } \alpha \in S^{(\hat{Z})},\ \beta \in S^{(\hat{Z})}, \text{ and } n \in \mathcal{N}. \tag{10}$$

[1]See [11, Chapter 3] for definitions and applications of the hitting and taboo probabilities for general Markov chains.

We focus on the taboo probabilities $\hat{t}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$ and $\hat{t}_{\mathrm{D}_r,\mathrm{D}_s}(n)$, which are given by

$$\begin{aligned}
\hat{t}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) &\stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \hat{\mathrm{E}}, \hat{Z}_2 \neq \hat{\mathrm{E}}, \cdots, \hat{Z}_{n-1} \neq \hat{\mathrm{E}}, \hat{Z}_n = \mathrm{D}_s \mid \hat{Z}_0 = \hat{\mathrm{E}}\right) \\
&= \Pr\left(\hat{Z}_1 = \mathrm{D}_1, \hat{Z}_2 = \mathrm{D}_2, \cdots, \hat{Z}_{n-1} = \mathrm{D}_{n-1}, \hat{Z}_n = \mathrm{D}_s \mid \hat{Z}_0 = \hat{\mathrm{E}}\right) \\
&= \begin{cases} u_1 \dfrac{u_2}{u_1} \dfrac{u_3}{u_2} \dfrac{u_4}{u_3} \cdots \dfrac{u_{s-1}}{u_{s-2}} \dfrac{u_s}{u_{s-1}}, & \text{for } s = n \in \mathcal{N}, \\ 0, & \text{otherwise,} \end{cases} \qquad \text{(from (2))} \\
&= \begin{cases} u_s, & \text{for } s = n \in \mathcal{N}, \\ 0, & \text{otherwise,} \end{cases} \qquad (11) \\
\hat{t}_{\mathrm{D}_r,\mathrm{D}_s}(n) &\stackrel{\triangle}{=} \Pr\left(\hat{Z}_1 \neq \hat{\mathrm{E}}, \hat{Z}_2 \neq \hat{\mathrm{E}}, \cdots, \hat{Z}_{n-1} \neq \hat{\mathrm{E}}, \hat{Z}_n = \mathrm{D}_s \mid \hat{Z}_0 = \mathrm{D}_r\right) \\
&= \Pr\left(\hat{Z}_1 = \mathrm{D}_{r+1}, \hat{Z}_2 = \mathrm{D}_{r+2}, \cdots, \hat{Z}_{n-1} = \mathrm{D}_{s-1}, \hat{Z}_n = \mathrm{D}_s \mid \hat{Z}_0 = \mathrm{D}_r\right) \\
&= \begin{cases} \dfrac{u_{r+1}}{u_r} \dfrac{u_{r+2}}{u_{r+1}} \dfrac{u_{r+3}}{u_{r+2}} \cdots \dfrac{u_{s-1}}{u_{s-2}} \dfrac{u_s}{u_{s-1}}, & \text{for } s = n \in \mathcal{N} \\ 0, & \text{otherwise,} \end{cases} \qquad \text{(from (2))} \\
&= \begin{cases} \dfrac{u_s}{u_r}, & \text{for } s = n \in \mathcal{N}, \\ 0, & \text{otherwise.} \end{cases} \qquad (12)
\end{aligned}$$

B. Calculation of element of n-step transition probability matrix $\{\boldsymbol{P}^{(\hat{Z})}\}^n$[10]

We denote the (α, β)th element of an n-step transition-probability matrix $\{\boldsymbol{P}^{(\hat{Z})}\}^n$ as $p^{(\hat{Z})}_{\alpha,\beta}(n)$. We define $p^{(\hat{Z})}_{\alpha,\beta}(0) = 1$ for $\alpha \in S^{(\hat{Z})}$ and $\beta \in S^{(\hat{Z})}$.

In the following, we derive the forms of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$, $p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n)$, $p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$, and $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$. Especially, we show that the form of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$ can be written as a recurrence formula and those of $p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n)$, $p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$, and $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$ can be expressed in the term of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(\cdot)$.

- Form of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$

  We classify all events that start in Ê at the initial epoch and end in state Ê at epoch $n(\geq 2)$ into a set of mutually exclusive events, of which each event that the first return to state Ê occurs at epoch i. From the Law of Total Probability and (9), $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$ is given by

$$\begin{aligned}
p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n) &= \sum_{i=1}^{n} \hat{h}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(i)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) \\
&= \sum_{i=1}^{n} \left(u_{i-1} - u_i\right) p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i), \qquad \text{(from (9))} \qquad \text{for } n \geq 2. \qquad (13)
\end{aligned}$$

  Note that (13) is valid for $n = 1$.

  Remark 1: From the definition, $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(0) = 1$, whereas (6) leads to $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(1) = 1 - u_1$. Hence, the values of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$ for $n = 2, 3, \cdots$ are available by solving (13) recursively with $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(0) = 1$ and $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(1) = 1 - u_1$. ■

- Form of $p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n)$

From the argument similar to the derivation of the form of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)$ and (8), $p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}$ can be written as:

$$\begin{aligned} p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}} &= \sum_{i=1}^{n} \hat{h}_{\mathrm{D}_r,\mathrm{E}}(i)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) \\ &= \frac{1}{u_r}\sum_{i=1}^{n}\Big(u_{r+i-1}-u_{r+n}\Big)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i), \qquad \text{(from (8))} \qquad \text{for } r\in\mathcal{N}. \end{aligned} \tag{14}$$

- Form of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$

  For the derivation of form of $p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$, we classify all events that start in Ê at the initial epoch and end in state D_s at epoch n by each event that the last return to state Ê prior to epoch n occurs at epoch $n-s$. If $s \le n$, we have

$$p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) = p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\, \hat{t}_{\mathrm{E},\mathrm{D}_s}(s). \tag{15}$$

  Substitution of (11) into (15) yields $p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n)$ given by

$$p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) = \begin{cases} u_s\, p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s), & \text{for } s\le n \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

- Form of $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$

  To derive the form of $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$, we consider two cases for paths from state D_r at the initial epoch to state D_s at epoch n: when more than one return to state Ê happens and when no return to state Ê happens.

  - When more than one return to state Ê happens

    In this case, we classify all events that start in D_r at the initial epoch and end in state D_s at epoch n by each event that the first return to state Ê occurs at epoch i and the last return to state Ê prior to epoch n occurs at epoch $n-i-s$. Then, using the hitting probability $\hat{h}_{\mathrm{D}_r,\hat{\mathrm{E}}}(i)$ and the taboo probability $\hat{t}_{\hat{\mathrm{E}},\mathrm{D}_s}(s)$, from (9) and (11), we have

$$\begin{aligned} p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) &= \sum_{i=1}^{n-s} \hat{h}_{\mathrm{D}_r,\hat{\mathrm{E}}}(i)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s)\, \hat{t}_{\hat{\mathrm{E}},\mathrm{D}_s}(s), \\ &= \frac{1}{u_r}\sum_{i=1}^{n-s}\Big(u_{r+i-1}-u_{r+i}\Big)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s)\, u_s. \end{aligned} \tag{17}$$

    Note that if $s \le n-1$ then the value of the transition probability $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$ in this case is positive.

  - When no return to state Ê happens

    In this case, the path from state D_r to state D_s is only $\mathrm{D}_r \to \mathrm{D}_{r+1} \to \mathrm{D}_{r+2} \to \cdots \mathrm{D}_{s-1} \to \mathrm{D}_s$. Then, using the taboo probability, from (12), we have

$$\begin{aligned} p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(s) &= \hat{t}_{\mathrm{D}_r,\mathrm{D}_s}(s) \\ &= \frac{u_s}{u_r}, \qquad \text{for } s = r+n. \end{aligned} \tag{18}$$

Note that the value of the transition probability $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$ in this case is positive if r, s, and n satisfy the following relation:

$$\begin{cases} s = r + n, \\ s \geq n + 1, \\ r \geq 1. \end{cases}$$

Hence, $p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$ is given by

$$p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) = \begin{cases} \dfrac{1}{u_r} \displaystyle\sum_{i=1}^{n-s} \Big(u_{r+i-1} - u_{r+i} \Big)\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s)\, u_s, & \text{for } s \leq n-1, \\ \dfrac{u_s}{u_r}, & \text{for } r \in \mathcal{N} \\ & \quad \text{and } s \geq n+1,\ s = r+n, \\ 0, & \text{otherwise}. \end{cases} \tag{19}$$

V. Calculation of n-step transition probabilities for Markov chain $\{Z_\kappa\}$

Let $\{\boldsymbol{P}^{(Z)}\}^n$ be an n-step transition-probability matrix for the packet-type MC $\{Z_\kappa\}$. We denote the $(\alpha,\ \beta)$th element of the matrix $\{\boldsymbol{P}^{(Z)}\}^n$ as $p^{(Z)}_{\alpha,\beta}(n)$. Subsequently, we derive the forms of $p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n)$, $p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n)$, $p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)$, and $p^{(Z)}_{\mathrm{E}_r,\mathrm{E}_s}(n)$, using the elements of $\{\boldsymbol{P}^{(\hat{Z})}\}^n$ derived above.

- Form of $p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n)$
  From the lumpability of $\boldsymbol{P}^{(Z)}$, $p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n)$ is given by

$$p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n) = p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n), \qquad \text{for } r \in \mathcal{N} \text{ and } s \in \mathcal{N}. \tag{20}$$

- Form of $p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n)$
  From the lumpability of $\boldsymbol{P}^{(Z)}$, $p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n)$ is given by

$$p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n) = p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n), \qquad \text{for } r \in \mathcal{N} \text{ and } s \in \mathcal{N}. \tag{21}$$

- Form of $p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)$
  - When $Z_n = \mathrm{E}_s (s \geq 2)$
    When $s \geq 2$, from the structure of the state-transition matrix $\boldsymbol{P}^{(Z)}$ shown in (2), the packet-type MC $\{Z_\kappa\}$ always visits state D_{s-1} at epoch $n-1$ if it visits state E_s at epoch n. Hence, $p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)$ is given by

$$\begin{aligned} p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n) &= p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_{s-1}}(n-1)\, p^{(Z)}_{\mathrm{D}_{s-1},\mathrm{E}_s}(1) \\ &= p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_{s-1}}(n-1)\, p^{(Z)}_{\mathrm{D}_{s-1},\mathrm{E}_s}(1) && \text{(from (20))} \\ &= p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_{s-1}}(n-1) \left(1 - \frac{u_s}{u_{s-1}} \right), \quad \text{for } s \geq 2 && \text{(from (2))}. \end{aligned} \tag{22}$$

  - When $Z_n = \mathrm{E}_1$

If the packet-type MC $\{Z_\kappa\}$ visits state E_1 at epoch n, a visit to any state E_m, $m \in \mathcal{N}$, always occurs at epoch $n-1$. Hence, we have

$$\begin{aligned} p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n) &= \sum_{m=1}^{\infty} p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{E}_m}(n-1)\, p^{(Z)}_{\mathrm{E}_m,\mathrm{E}_1}(1) \\ &= p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n-1)\left(1-u_1\right), \qquad \text{for } r \in \mathcal{N} \text{ and } s = 1 \qquad \text{(from (2))}. \end{aligned} \tag{23}$$

Therefore, $p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)$ is given by

$$p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n) = \begin{cases} p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_{s-1}}(n-1)\left(1-\dfrac{u_s}{u_{s-1}}\right), & \text{for } r \in \mathcal{N} \text{ and } s \geq 2, \\ p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n-1)\left(1-u_1\right), & \text{for } r \in \mathcal{N} \text{ and } s = 1. \end{cases} \tag{24}$$

- Form of $p^{(Z)}_{\mathrm{E}_r,\mathrm{E}_s}(n)$
From the argument similar to the derivation of form of $p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)$, this n-step transition probability is given by

$$p^{(Z)}_{\mathrm{E}_r,\mathrm{E}_s}(n) = \begin{cases} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_{s-1}}(n-1)\left(1-\dfrac{u_s}{u_{s-1}}\right), & \text{for } r \in \mathcal{N} \text{ and } s \geq 2, \\ p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\left(1-u_1\right), & \text{for } r \in \mathcal{N} \text{ and } s = 1. \end{cases} \tag{25}$$

VI. Derivation of form of covariance of $X_0^{(p)}$ and $X_n^{(p)}$

In the following, we assume that the stochastic process $\{X_\kappa^{(p)}\}$ is *stationary*. In this case, the covariance of random variables of $X_0^{(p)}$ and $X_n^{(p)}$, denoted as $\mathrm{Cov}[X_0^{(p)}, X_n^{(p)}]$, is given by

$$\begin{aligned} \mathrm{Cov}\left[X_0^{(p)}, X_n^{(p)}\right] &\triangleq E\left[\left(X_0^{(p)} - E\left[X_0^{(p)}\right]\right)\left(X_n^{(p)} - E\left[X_n^{(p)}\right]\right)\right] \\ &= E\left[X_0^{(p)} \cdot X_n^{(p)}\right] - \left\{\ell^{(p)}\right\}^2, \end{aligned} \tag{26}$$

where $\ell^{(p)}$ is the mean packet size i.e., $\ell^{(p)} = E[X_\kappa^{(p)}]$ for any $\kappa \in \mathcal{N}_0$. Subsequently, we derive analytical forms of $\ell^{(p)}$ and $E[X_0^{(p)} \cdot X_n^{(p)}]$ in (26).

A. *Derivation of form of mean packet size $\ell^{(p)}$*

1) *Stationary-state probabilities for packet-type Markov chain $\{Z_\kappa\}$*

We denote the stationary-state probabilities for the MC $\{Z_\kappa\}$ by $\pi^{(\mathrm{D}_r)}$ and $\pi^{(\mathrm{E}_s)}$, respectively. They are given by

$$\pi^{(\mathrm{D}_r)} \triangleq \Pr\left(Z_\kappa = \mathrm{D}_r\right) = \Delta^{-1} u_r, \qquad \text{for } r \in \mathcal{N}, \tag{27}$$

$$\pi^{(\mathrm{E}_s)} \triangleq \Pr\left(Z_\kappa = \mathrm{E}_s\right) = \Delta^{-1}(u_{s-1} - u_s), \qquad \text{for } s \in \mathcal{N}, \tag{28}$$

where

$$\Delta \triangleq \sum_{s=0}^{\infty} u_s. \tag{29}$$

2) Conditional expectation for stochastic process $\{X_\kappa^{(p)}\}$

Let $F^{(\alpha)}(\cdot)$ be the conditional distribution function of $X_\kappa^{(p)}$ given a state of Z_κ is $\alpha \in S^{(Z)}$. Then, we have

$$\begin{aligned}
F^{(\mathrm{D}_r)}(x) &\triangleq \Pr\left(X_\kappa^{(p)} \le x \,|\, Z_\kappa = \mathrm{D}_r\right) \\
&= \mathbf{1}(x - \ell_d), \qquad\qquad \text{for } r \in \mathcal{N}, \qquad (30) \\
F^{(\mathrm{E}_s)}(x) &\triangleq \Pr\left(X_\kappa^{(p)} \le x \,|\, Z_\kappa = \mathrm{E}_s\right) \\
&= \frac{\Pr\Big((s-1)\,\ell_d < X_i^{(m)} \le x + (s-1)\,\ell_d\Big)}{\Pr\Big((s-1)\ell_d < X_i^{(m)} \le s\,\ell_d\Big)} \\
&= \begin{cases} 0, & \text{if } x \le 0, \\ \dfrac{F^{(m)}(x + (s-1)\,\ell_d) - F^{(m)}((s-1)\,\ell_d)}{u_{s-1} - u_s}, & \text{if } 0 < x \le \ell_d, \\ 1, & \text{if } x > \ell_d, \end{cases} \\
&\qquad\qquad \text{for } s \in \mathcal{N}. \qquad (31)
\end{aligned}$$

where an indicator function $\mathbf{1}(x)$ is defined as

$$\mathbf{1}(x) \triangleq \begin{cases} 1, & \text{if } x \ge 0, \\ 0, & \text{if } x < 0. \end{cases}$$

We denote conditional expectation for $\{X_\kappa^{(p)}\}$ given that $Z_\kappa = \alpha \in S^{(Z)}$, i.e., $E[X_\kappa^{(p)} \,|\, Z_\kappa = \alpha]$ by $m^{(\alpha)}$. Then, we have

$$\begin{aligned}
m^{(\mathrm{D}_r)} &\triangleq E\left[X_\kappa^{(p)} \,|\, Z_\kappa = \mathrm{D}_r\right] \\
&= \int_0^\infty x\, dF^{(\mathrm{D}_r)}(x) \\
&= \ell_d, \qquad \text{(from (30))} \qquad \text{for } r \in \mathcal{N}, \quad (32) \\
m^{(\mathrm{E}_s)} &\triangleq E\left[X_\kappa^{(p)} \,|\, Z_\kappa = \mathrm{E}_s\right] \\
&= \int_0^\infty x\, dF^{(\mathrm{E}_s)}(x) \\
&= \int_0^{\ell_d} \frac{x\, dF^{(m)}(x + (s-1)\,\ell_d)}{u_{s-1} - u_s} \qquad \text{(from (31))} \\
&= \frac{1}{u_{s-1} - u_s} \int_{(s-1)\,\ell_d}^{s\,\ell_d} \left(x - (s-1)\,\ell_d\right)\, dF^{(m)}(x) \\
&= \frac{1}{u_{s-1} - u_s} \left\{ \int_{(s-1)\,\ell_d}^{s\,\ell_d} x\, dF^{(m)}(x) - \int_{(s-1)\,\ell_d}^{s\,\ell_d} (s-1)\,\ell_d\, dF^{(m)}(x) \right\} \\
&= \frac{v_{s-1} - v_s}{u_{s-1} - u_s} - (s-1)\,\ell_d, \qquad \text{for } s \in \mathcal{N}, \quad (33)
\end{aligned}$$

where

$$v_r \triangleq \int_{r\ell_d}^\infty x\, dF^{(m)}(x), \qquad \text{for } r \in \mathcal{N}, \qquad (34)$$

with

$$v_0 = \int_0^\infty x\, dF^{(m)}(x) = \ell^{(m)}. \qquad (35)$$

3) Mean packet size $\ell^{(p)}$

The mean packet size $\ell^{(p)}$ is given by

$$\begin{aligned}
\ell^{(p)} &\triangleq E\left[X_\kappa^{(p)}\right] \\
&= \sum_{r=1}^{\infty} \pi^{(\mathrm{D}_r)} \ell_d + \sum_{s=1}^{\infty} \pi^{(\mathrm{E}_s)} m^{(\mathrm{E}_s)} \\
&= \Delta^{-1}\, \ell_d \sum_{r=1}^{\infty} u_r + \Delta^{-1} \sum_{s=1}^{\infty} (u_{s-1} - u_s)\, m^{(\mathrm{E}_s)} && \text{(from (27) and (28))} \\
&= \Delta^{-1}\, \ell_d\, (\Delta - 1) && \text{(from (29))} \\
&\quad + \Delta^{-1} \left(\ell^{(m)} + \ell_d - \Delta\, \ell_d\right) \\
&= \Delta^{-1}\, \ell^{(m)}, && (36)
\end{aligned}$$

using

$$\begin{aligned}
\sum_{s=1}^{\infty} (u_{s-1} - u_s)\; m^{(\mathrm{E}_s)} &= \sum_{s=1}^{\infty} (u_{s-1} - u_s)\, \frac{v_{s-1} - v_s - (s-1)\, \ell_d\, (u_{s-1} - u_s)}{u_{s-1} - u_s} \\
&= v_0 - \ell_d \sum_{s=1}^{\infty} (s-1)\, (u_{s-1} - u_s) \\
&= \ell^{(m)} - \ell_d \left(\sum_{s=0}^{\infty} u_s - u_0\right) && \text{(from (35))} \\
&= \ell^{(m)} + \ell_d - \Delta\, \ell_d && \text{(from (4) and (29)).} \quad (37)
\end{aligned}$$

Note that another derivation of (36) can be found in [8].

B. Derivation of form of $E[X_0^{(p)} \cdot X_n^{(p)}]$

The first term in (26), $E[X_0^{(p)} \cdot X_n^{(p)}]$, can be expressed as:

$$\begin{aligned}
E\left[X_0^{(p)} \cdot X_n^{(p)}\right] &= \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_0 = \mathrm{D}_r, Z_n = \mathrm{D}_s\right] \Pr\left(Z_0 = \mathrm{D}_r, Z_n = \mathrm{D}_s\right) \\
&\quad + \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_0 = \mathrm{D}_r, Z_n = \mathrm{E}_s\right] \Pr\left(Z_0 = \mathrm{D}_r, Z_n = \mathrm{E}_s\right) \\
&\quad + \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_0 = \mathrm{E}_r, Z_n = \mathrm{D}_s\right] \Pr\left(Z_0 = \mathrm{E}_r, Z_n = \mathrm{D}_s\right) \\
&\quad + \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_0 = \mathrm{E}_r, Z_n = \mathrm{E}_s\right] \Pr\left(Z_0 = \mathrm{E}_r, Z_n = \mathrm{E}_s\right) \\
&\triangleq A_1 + A_2 + A_3 + A_4. && (38)
\end{aligned}$$

1) Preliminary

Before derivation of each term in (38), we calculate terms $\sum_{r=1}^{\infty}(u_{r+i-1} - u_{r+i})$, $\sum_{r=1}^{\infty} u_r\, p_{\mathrm{D}_r,\mathrm{E}}^{(\hat{Z})}(n)$, and $\sum_{r=1}^{\infty} u_r\, p_{\mathrm{D}_r,\mathrm{D}_s}^{(\hat{Z})}(n)$.

- Form of $\sum_{r=1}^{\infty}(u_{r+i-1} - u_{r+i})$

$$\sum_{r=1}^{\infty} (u_{r+i-1} - u_{r+i}) = u_i - u_{i+1} + u_{i+1} - u_{i+2} + \cdots$$
$$= u_i. \tag{39}$$

- Form of $\sum_{r=1}^{\infty} u_r \, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{E}}(n)$

$$\begin{aligned}
\sum_{r=1}^{\infty} u_r \, p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n) &= \sum_{r=1}^{\infty} u_r \frac{1}{u_r} \sum_{i=1}^{n} \Big(u_{r+i-1} - u_{r+i} \Big) \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) && \text{(from (14))} \\
&= \sum_{i=1}^{n} \left\{ \sum_{r=1}^{\infty} \Big(u_{r+i-1} - u_{r+i} \Big) \right\} p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) \\
&= \sum_{i=1}^{n} u_i \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) && \text{(from (39))} \\
&= 1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n). && (40)
\end{aligned}$$

For the derivation of (40), we used

$$\begin{aligned}
1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n) &= \sum_{s=1}^{\infty} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) && (41a) \\
&= \sum_{s=1}^{n-s} u_s \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i) \\
&\qquad \text{(because } p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) = u_s \, p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s) \text{ for } n \geq s \text{ from (16)),} && (41b) \\
&= \sum_{s=1}^{n} u_s \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s) \\
&\qquad \text{(because } p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) = 0 \text{ for } n < s \text{ from (16) additionally),} && (41c)
\end{aligned}$$

because of the transition-probability-matrix property, that is $p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n) + \sum_{s=1}^{\infty} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) = 1$.

- Form of $\sum_{r=1}^{\infty} u_r \, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n)$
  From (19), we consider the following three cases: $s \leq n-1$, $s = n$, and $s \geq n$
  - When $s \leq n-1$

$$\begin{aligned}
\sum_{r=1}^{\infty} u_r \, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) &= \sum_{r=1}^{\infty} u_r \frac{1}{u_r} \sum_{i=1}^{n-s} \Big(u_{r+i-1} - u_{r+i} \Big) \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s) \, u_s && \text{(from (19))} \\
&= \sum_{i=1}^{n-s} \left\{ \sum_{r=1}^{\infty} \Big(u_{r+i-1} - u_{r+i} \Big) \right\} p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s) \, u_s \\
&= \sum_{i=1}^{n-s} u_i \, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-i-s) \, u_s && \text{(from (39))} \\
&= u_s \, \Big(1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s) \Big), \qquad \text{for } s \leq n-1. && (42)
\end{aligned}$$

  - When $s = n$

$$\sum_{r=1}^{\infty} u_r\, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) = 0, \qquad \text{for } s = n. \tag{43}$$

– When $s \geq n+1$

$$\begin{aligned}
\sum_{r=1}^{\infty} u_r\, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) &= u_{s-n}\, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) && \text{(because } s = r+n \text{ from (19))} \\
&= u_{s-n}\, \frac{u_s}{u_{s-n}} && \text{(because } p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) = u_s/u_{s-n} \text{ from (19))} \\
&= u_s, && \text{for } s \geq n+1. \qquad (44)
\end{aligned}$$

Therefore, we have

$$\sum_{r=1}^{\infty} u_r\, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_s}(n) = \begin{cases} u_s \left(1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\right), & \text{for } s \leq n-1, \\ 0, & \text{for } s = n, \\ u_s, & \text{for } s \geq n+1. \end{cases} \tag{45}$$

2) Calculation of terms in (38)

We calculate of each term in (38) as follows.

- Form of term A_1

$$\begin{aligned}
A_1 &\triangleq \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_0 = \mathrm{D}_r, Z_n = \mathrm{D}_s\right] \Pr\left(Z_0 = \mathrm{D}_r, Z_n = \mathrm{D}_s\right) \\
&= \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} {\ell_d}^2\, \pi^{(\mathrm{D}_r)}\, p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n) \\
&= \Delta^{-1}\, {\ell_d}^2 \sum_{s=1}^{\infty}\sum_{r=1}^{\infty} u_r\, p^{(Z)}_{\mathrm{D}_r,\mathrm{D}_s}(n) && \text{(from (27))} \\
&= \Delta^{-1}\, {\ell_d}^2 \left\{\sum_{s=1}^{n-1} u_s \left(1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\right) + \sum_{s=n+1}^{\infty} u_s\right\} && \text{(from (45))} \\
&= \Delta^{-1}\, {\ell_d}^2 \left\{\sum_{s=1}^{n} u_s \left(1 - p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\right) + \sum_{s=n+1}^{\infty} u_s\right\} && \text{(from } p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(0) = 1\text{)} \\
&= \Delta^{-1}\, {\ell_d}^2 \left\{\sum_{s=1}^{\infty} u_s - \sum_{s=1}^{n} u_s\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\right\} \\
&= \Delta^{-1}\, {\ell_d}^2 \left\{\sum_{s=0}^{\infty} u_s - u_0 - 1 + p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)\right\} && \text{(from (41c))} \\
&= \Delta^{-1}\, {\ell_d}^2 \left(\Delta - 2 + p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)\right) && \text{(from (4) and (29)).} \qquad (46)
\end{aligned}$$

- Form of term A_2

$$
\begin{aligned}
A_2 &\stackrel{\triangle}{=} \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_1 = \mathrm{D}_r, Z_s = \mathrm{E}_s\right] \Pr\left(Z_1 = \mathrm{D}_r, Z_s = \mathrm{E}_s\right) \\
&= \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \ell_d\, m^{(\mathrm{E}_s)}\, \pi^{(\mathrm{D}_r)}\, p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n) \\
&= \Delta^{-1}\,\ell_d \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} u_r\, p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)\, m^{(\mathrm{E}_s)} && \text{(from (27))} \\
&= \Delta^{-1}\,\ell_d \left\{ \sum_{r=1}^{\infty} u_r\, p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_1}(n)\, m^{(\mathrm{E}_s)} + \sum_{r=1}^{\infty}\sum_{s=2}^{\infty} p^{(Z)}_{\mathrm{D}_r,\mathrm{E}_s}(n)\, m^{(\mathrm{E}_s)} \right\} \\
&= \Delta^{-1}\,\ell_d \left\{ \sum_{r=1}^{\infty} u_r\, p^{(\hat{Z})}_{\mathrm{D}_r,\hat{\mathrm{E}}}(n-1) \left(1 - u_1\right) m^{(\mathrm{E}_1)} \right. \\
&\qquad \left. + \sum_{s=2}^{\infty}\sum_{r=1}^{\infty} u_r\, p^{(\hat{Z})}_{\mathrm{D}_r,\mathrm{D}_{s-1}}(n-1) \left(\frac{u_{s-1} - u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)} \right\} && \text{(from (14))} \\
&= \Delta^{-1}\,\ell_d \left\{ \left(1 - p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\right) \left(1 - u_1\right) m^{(\mathrm{E}_1)} \right. && \text{(from (40))} \\
&\qquad + \sum_{s=2}^{n-1} u_{s-1} \left(1 - p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\right) \left(\frac{u_{s-1} - u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)} && \text{(from case for } s \leq n-1 \text{ in (45))} \\
&\qquad \left. + \sum_{s=n+1}^{\infty} u_{s-1} \left(\frac{u_{s-1} - u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)} \right\} && \text{(from case for } s \geq n+1 \text{ in (45))} \\
&= \Delta^{-1}\,\ell_d \left\{ \sum_{s=1}^{\infty} \left(u_{s-1} - u_s\right) m^{(\mathrm{E}_s)} - \sum_{s=1}^{n} p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s) \left(u_{s-1} - u_s\right) m^{(\mathrm{E}_s)} \right\} \\
&= \Delta^{-1}\,\ell_d \left(\ell^{(m)} + \ell_d - \Delta\,\ell_d\right) \\
&\qquad - \Delta^{-1}\,\ell_d \sum_{s=1}^{n} p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s) \left(u_{s-1} - u_s\right) m^{(\mathrm{E}_s)} && \text{(from (37)).} \qquad (47)
\end{aligned}
$$

- Form of term A_3

$$
\begin{aligned}
A_3 &\stackrel{\triangle}{=} \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} E\left[\left(X_0^{(p)} \cdot X_n^{(p)}\right) \mid Z_1 = \mathrm{E}_r, Z_s = \mathrm{D}_s\right] \Pr\left(Z_1 = \mathrm{E}_r, Z_s = \mathrm{D}_s\right) \\
&= \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} m^{(\mathrm{E}_r)}\, \ell_d\, \pi^{(\mathrm{E}_r)}\, p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n) \\
&= \Delta^{-1}\,\ell_d \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \left(u_{r-1} - u_r\right) m^{(\mathrm{E}_r)}\, p^{(Z)}_{\mathrm{E}_r,\mathrm{D}_s}(n) && \text{(from (28))} \\
&= \Delta^{-1}\,\ell_d \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \left(u_{r-1} - u_r\right) m^{(\mathrm{E}_r)}\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) && \text{(from (16))} \\
&= \Delta^{-1}\,\ell_d \left\{ \sum_{r=1}^{\infty} \left(u_{r-1} - u_r\right) m^{(\mathrm{E}_r)} \right\} \left\{ \sum_{s=1}^{\infty} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_s}(n) \right\} \\
&= \Delta^{-1}\,\ell_d \left(\ell^{(m)} + \ell_d - \Delta\,\ell_d\right) \left(1 - p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)\right) && \text{(from (37)).} \qquad (48)
\end{aligned}
$$

- Form of term A_4

$$
\begin{aligned}
A_4 &\triangleq \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} E\left[\left(X_0^{(p)}\cdot X_n^{(p)}\right) \mid Z_1=\mathrm{E}_r, Z_s=\mathrm{E}_s\right] \Pr\left(Z_1=\mathrm{E}_r, Z_s=\mathrm{E}_s\right)\\
&= \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} m^{(\mathrm{E}_r)}\, m^{(\mathrm{E}_s)}\, \pi^{(\mathrm{E}_r)}\, p^{(Z)}_{\mathrm{E}_r,\mathrm{E}_s}(n)\\
&= \sum_{r=1}^{\infty} \pi^{(\mathrm{E}_r)}\, m^{(\mathrm{E}_r)}\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\left(1-u_1\right) m^{(\mathrm{E}_1)}\\
&\quad + \sum_{r=1}^{\infty}\sum_{s=2}^{\infty} \pi^{(\mathrm{E}_r)}\, m^{(\mathrm{E}_r)}\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_{s-1}}(n-1)\left(\frac{u_{s-1}-u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)} \qquad \text{(from (25))}\\
&= \left\{\sum_{r=1}^{\infty} \pi^{(\mathrm{E}_r)}\, m^{(\mathrm{E}_r)}\right\}\\
&\qquad \times \left\{p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\left(1-u_1\right) m^{(\mathrm{E}_1)} + \sum_{s=2}^{\infty} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_{s-1}}(n-1)\left(\frac{u_{s-1}-u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)}\right\}\\
&= \left\{\Delta^{-1}\sum_{r=1}^{\infty}\left(u_{r-1}-u_r\right) m^{(\mathrm{E}_r)}\right\} \qquad \text{(from (28))}\\
&\qquad \times \left\{p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\left(1-u_1\right) m^{(\mathrm{E}_1)} + \sum_{s=2}^{\infty} p^{(\hat{Z})}_{\hat{\mathrm{E}},\mathrm{D}_{s-1}}(n-1)\left(\frac{u_{s-1}-u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)}\right\}\\
&= \left\{\Delta^{-1}\sum_{r=1}^{\infty}\left(u_{r-1}-u_r\right) m^{(\mathrm{E}_r)}\right\}\\
&\qquad \times \left\{p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-1)\left(1-u_1\right) m^{(\mathrm{E}_1)} + \sum_{s=2}^{n} u_{s-1}\, p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\left(\frac{u_{s-1}-u_s}{u_{s-1}}\right) m^{(\mathrm{E}_s)}\right\} \quad \text{(from (16))}\\
&= \Delta^{-1}\left(\ell^{(m)}+\ell_d-\Delta\,\ell_d\right)\left\{\sum_{s=1}^{n} p^{(Z)}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\left(u_{s-1}-u_s\right) m^{(\mathrm{E}_s)}\right\} \qquad \text{(from (37)).}
\end{aligned}
\tag{49}
$$

3) Final form of $E[X_0^{(p)}\cdot X_n^{(p)}]$

Substitution of (46) to (49) into (38) yields the final form of $E[X_0^{(p)}\cdot X_n^{(p)}]$, which is given by

$$
\begin{aligned}
E\left[X_0^{(p)}\cdot X_n^{(p)}\right] &= A_1 + A_2 + A_3 + A_4\\
&= \Delta^{-1}\,\ell_d\,\ell^{(m)}\\
&\quad - \left(\ell_d - \Delta^{-1}\,\ell^{(m)}\right)\\
&\qquad \times \left\{\ell_d\left(1-p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n)\right) + \sum_{s=1}^{n} p^{(\hat{Z})}_{\hat{\mathrm{E}},\hat{\mathrm{E}}}(n-s)\,\left(u_{s-1}-u_s\right) m^{(\mathrm{E}_s)}\right\}.
\end{aligned}
\tag{50}
$$

From (36), (50) can be written as:

$$E\left[X_0^{(p)} \cdot X_n^{(p)}\right] =$$
$$\ell_d\,\ell^{(p)} \quad - \left(\ell_d - \ell^{(p)}\right)\left\{\ell_d\,\left(1 - p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n)\right) + \sum_{s=1}^{n} p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n-s)\,\left(u_{s-1} - u_s\right)\,m^{(\mathrm{E}_s)}\right\}. \quad (51)$$

Example 1: Case of exponential message size distribution. Assuming that message sizes are exponentially distributed with mean $\ell^{(m)}$:

$$F^{(m)}(x) = \begin{cases} 1 - e^{-\frac{x}{\ell^{(m)}}}, & \text{if } x > 0, \\ 0, & \text{if } x \leq 0. \end{cases} \quad (52)$$

In this case, the terms in (51) u_s, $p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n)$, and $m^{(\mathrm{E}_s)}$ are given by

$$u_s = u^s, \qquad \text{for } s \in \mathcal{N}, \quad (53)$$
$$p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n) = \begin{cases} 1 - u, & \text{for } n \in \mathcal{N}, \\ 1, & \text{for } n = 1, \end{cases} \quad (54)$$
$$m^{(\mathrm{E}_s)} = \frac{(1-u)\,\ell^{(m)} - u\,\ell_d}{1-u}, \qquad \text{for } s \in \mathcal{N}, \quad (55)$$

with $u \triangleq e^{-\frac{\ell_d}{\ell^{(m)}}}$. Hence, from (53) to (55), the last term in (51) is given by

$$\begin{aligned}\sum_{s=1}^{n} p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n-s)\,\left(u_{s-1} - u_s\right)\,m^{(\mathrm{E}_s)} &= \sum_{s=1}^{n-1} p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n-s)\,\left(u_{s-1} - u_s\right)\,m^{(\mathrm{E}_s)} \\ &\quad + p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(0)\,\left(u_{n-1} - u_n\right)\,m^{(\mathrm{E}_n)} \\ &= (1-u)\ell^{(m)} - u\,\ell_d. \end{aligned} \quad (56)$$

Substitution of (54) and (56) into (51) yields

$$E\left[X_0^{(p)} \cdot X_n^{(p)}\right] = \left\{\ell^{(p)}\right\}^2, \quad (57)$$

resulting in $\mathrm{Cov}[X_0^{(p)}, X_n^{(p)}]$ being zero for any $n \in \mathcal{N}$. Thus, random variables of $X_0^{(p)}$ and $X_n^{(p)}$ for any $n \in \mathcal{N}$ are uncorrelated, even though message segmentations happen. The reason for this is due to memoryless property of an exponential distribution. Note that the random variables of $X_0^{(p)}$ and $X_n^{(p)}$ are not independent.[2] ■

Example 2: When payload size is very large. In this case, we have

$$u_r \approx \begin{cases} 0, & \text{for } r \geq 1, \\ 1, & \text{for } r = 0, \end{cases} \quad (58)$$
$$p_{\hat{\mathrm{E}},\hat{\mathrm{E}}}^{(\hat{Z})}(n) \approx \begin{cases} 0, & \text{for } n \geq 1, \\ 1, & \text{for } n = 0, \end{cases} \quad (59)$$
$$m^{(\mathrm{E}_s)} \approx \ell^{(m)}, \quad (60)$$
$$\ell^{(p)} \approx \ell^{(m)}. \quad (61)$$

Therefore, $E[X_0^{(p)} \cdot X_n^{(p)}]$ is approximately given by $\{\ell^{(m)}\}^2$, resulting in $\mathrm{Cov}[X_0^{(p)}, X_n^{(p)}]$ being zero for any $n \in \mathcal{N}$. The reason for this is because the message sizes are assumed to be *i.i.d.*

[2]Not all uncorrelated random variables are independent [12, p. 51].

random variables and almost no message segmentation happen, resulting in almost all packets corresponding to the respective messages. ■

Example 3: When payload size is very small. In this case, $\ell^{(p)}$ can be approximated by ℓ_d. Hence, $E[X_0^{(p)} \cdot X_n^{(p)}]$ is approximately equal to zero because $E[X_0^{(p)} \cdot X_n^{(p)}] \approx {\ell_d}^2$. As a result, $\mathrm{Cov}[X_0^{(p)}, X_n^{(p)}]$ is almost zero for any $n \in \mathcal{N}$. ■

VII. Numerical results and discussions

To investigate the effect of the payload size ℓ_d on the correlation of packet sizes, we introduce a correlation coefficient of $X_0^{(p)}$ and $X_n^{(p)}$, denoted by $\mathrm{Corr}(n)$, which is defined as:

$$\mathrm{Corr}(n) \triangleq \frac{\mathrm{Cov}\left[X_0^{(p)}, X_n^{(p)}\right]}{\left\{\sigma^{(p)}\right\}^2}, \tag{62}$$

where $\{\sigma^{(p)}\}^2$ is a variance of the packet sizes (see Appendix for the derivation of $\{\sigma^{(p)}\}^2$).

Suppose that message sizes are lognormally distributed (such as HTTP messages, see [7]):

$$F^{(m)}(x) = \begin{cases} \displaystyle\int_{y=0}^{y=x} \frac{1}{\sqrt{2\pi}\sigma y} e^{\frac{-(\log y - \mu)^2}{2\sigma^2}} dy, & \text{if } x > 0, \\ 0, & \text{if } x \le 0, \end{cases} \tag{63}$$

In this case, we cannot obtain the explicit analytical forms of u_i and $m^{(\mathrm{E}_s)}$. Hence, we need to calculate their values numerically. For the calculation of $m^{(\mathrm{E}_s)}$, we may use $dF^{(\mathrm{E}_s)}(x)$ given by

$$dF^{(\mathrm{E}_s)}(x) = \begin{cases} 0, & \text{if } x \le 0, \\ \dfrac{dF^{(m)}((s-1)\,\ell_d + x)}{u_{s-1} - u_s}, & \text{if } 0 < x \le \ell_d, \\ 0, & \text{if } x > \ell_d, \end{cases} \qquad \text{(from (31))}$$

$$= \begin{cases} 0, & \text{if } x \le 0, \\ \dfrac{e^{-\frac{\left\{\log\left((s-1)\,\ell_d + x\right) - \mu\right\}^2}{2\,\sigma^2}}}{\sqrt{2\,\pi}\sigma\left\{(s-1)\,\ell_d + x\right\}(u_{s-1} - u_s)}, & \text{if } 0 < x \le \ell_d, \\ 0, & \text{if } x > \ell_d. \end{cases} \qquad \text{(from (63))} \tag{64}$$

As shown in the three-dimensional plot in Fig. 1, the correlation coefficient (or auto-correlation function) $\mathrm{Corr}(n)$ are plotted versus payload sizes ℓ_d and lags n. The distribution parameters μ and σ in (63) are assumed to be 6.34 and 2.07, respectively, based on the measured mean message size $\ell^{(m)} = 4,827$ bytes and the measured standard deviation $\sigma^{(m)} = 41,008$ bytes [7]. From this figure, we observe that

- the packet sizes are little correlated when the payload sizes ℓ_d is enough large because few message-segmentations happen, or ℓ_d is enough small because almost messages are segmented, resulting in packet sizes being almost ℓ_d.
- they are highly correlated when ℓ_d are around $10,000$ bytes and lags n are small.

Figure 2 shows the correlation coefficient $\mathrm{Corr}(n)$ versus lags n for different payload sizes ℓ_d. In this figure, the payload sizes ℓ_d of 536, 1460, 2272 bytes are used. Note that 1460 bytes is a payload size used commonly for TCP[13]. From this figure plotting $\log_{10} \mathrm{Corr}(n)$ against $\log_{10} n$, we find that the packet sizes exhibit heavily correlations across large n with payload sizes used commonly.

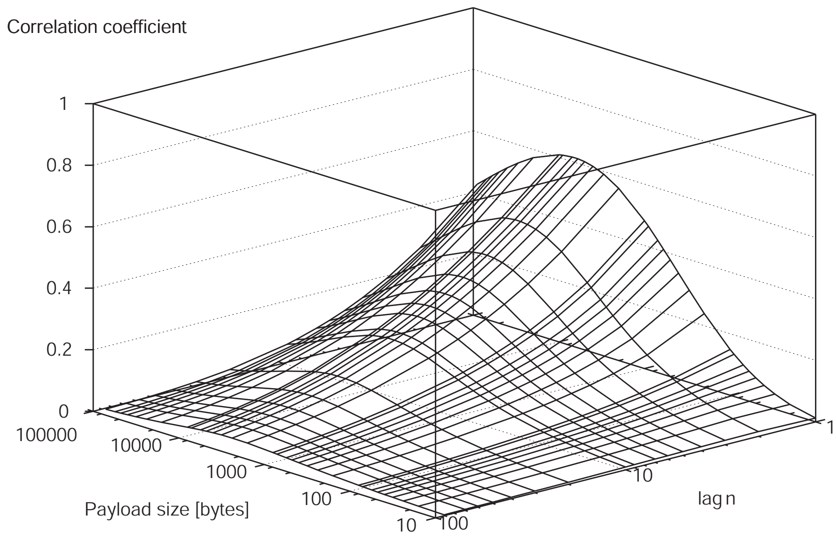

Fig. 1. Correlation coefficient $\mathrm{Corr}(n)$ versus lags n and payload sizes ℓ_d.

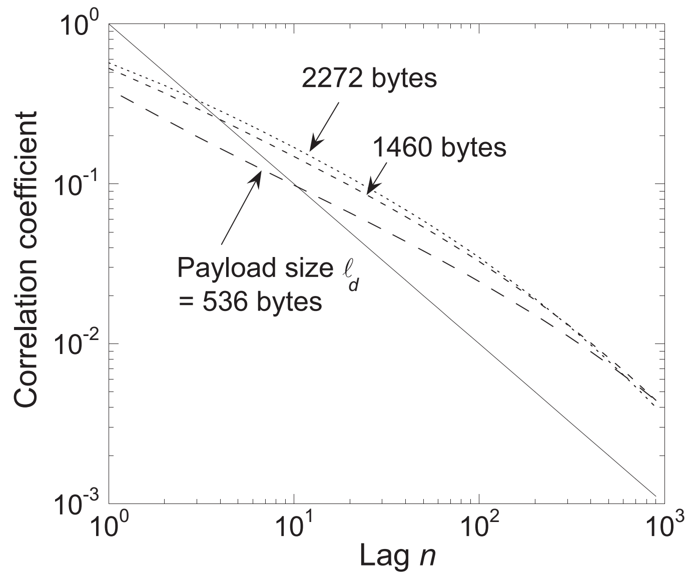

Fig. 2. Correlation coefficient $\mathrm{Corr}(n)$ versus lags n for different payload sizes ℓ_d (straight line corresponds to $\mathrm{Corr}(n) = n^{-1}$).

VIII. Conclusion

In this report, we derived an analytical form of covariance of a packet size sequence for an environment where message segmentations happen. To derive the form of the covariance efficiently, we introduced the Markov chain representing a sequence of packet types (implying body and edge packets), and the aggregated Markov chain using the lumpability. Using the analytical form, we showed that packet sizes are uncorrelated, i.e., the covariance is always zero, when message sizes are exponentially distributed because of the memoryless property of an exponential distribution, even though the message segmentations happen. However, numerical results where HTTP messages are lognormally distributed according to an actual traffic measurement, we demonstrated that TCP-packet sizes exhibit the heavily-correlated property in cases of payload sizes used commonly such as 1460 bytes.

Appendix

Derivation of packet size variance $\{\sigma^{(p)}\}^2$[10]

A. Preliminaries

To calculate the packet size variance $\{\sigma^{(p)}\}^2$ efficiently, we first calculate several terms $\sum_{s=1}^{\infty}\int_{(s-1)\,\ell_d}^{s\ell_d} x^2\,dF^{(m)}(x)$, $\sum_{s=1}^{\infty}\int_{(s-1)\,\ell_d}^{s\ell_d} x^2\,dF^{(m)}(x)$, and $\sum_{s=1}^{\infty}(s-1)^2\int_{(s-1)\,\ell_d}^{s\,\ell_d} dF^{(m)}(x)$ as follows.

- Form of $\sum_{s=1}^{\infty}\int_{(s-1)\,\ell_d}^{s\ell_d} x^2\,dF^{(m)}(x)$

$$\begin{aligned}\sum_{s=1}^{\infty}\int_{(s-1)\,\ell_d}^{s\ell_d} x^2\,dF^{(m)}(x) &= \int_0^{\infty} x^2\,dF^{(m)}(x)\\ &= \left\{\sigma^{(m)}\right\}^2+\left\{\ell^{(m)}\right\}^2. \end{aligned}\tag{65}$$

- Form of $\sum_{s=1}^{\infty}(s-1)\int_{(s-1)\,\ell_d}^{s\,\ell_d} x\,dF^{(m)}(x)$

$$\begin{aligned}\sum_{s=1}^{\infty}(s-1)\int_{(s-1)\,\ell_d}^{s\,\ell_d} x\,dF^{(m)}(x) &= \sum_{s=1}^{\infty}(s-1)\,(v_s-v_{s-1})\\ &= v_1-v_2+2\,(v_2-v_3)+3\,(v_3-v_4)+\cdots\\ &= v_1+v_2+v_3+v_4+\cdots\\ &= \sum_{s=0}^{\infty}v_s-v_0\\ &= \sum_{s=0}^{\infty}v_s-\ell^{(m)} \qquad \text{(from (35))}. \end{aligned}\tag{66}$$

- Form of $\sum_{s=1}^{\infty}(s-1)^2\int_{(s-1)\,\ell_d}^{s\,\ell_d} dF^{(m)}(x)$

$$\begin{aligned}\sum_{s=1}^{\infty}(s-1)^2\int_{(s-1)\,\ell_d}^{s\,\ell_d} dF^{(m)}(x) &= \sum_{s=1}^{\infty}(s-1)^2\,\left(u_{s-1}-u_s\right)\\ &= \sum_{s=1}^{\infty}s^2\,\left(u_{s-1}-u_s\right)-2\sum_{s=1}^{\infty}s\,\left(u_{s-1}-u_s\right)+1\\ &= u_0-u_1+2^2\,(u_1-u_2)+3^2\,(u_2-u_3)\cdots\\ &\quad -2\left\{u_0-u_1+2\,(u_1-u_2)+3\,(u_2-u_3)+\cdots+\right\}+1\\ &= \left(1^2-0^2\right)\,u_0+\left(2^2-1^2\right)\,u_1+\left(3^2-2^2\right)\,u_2+\left(4^2-3^2\right)\,u_3+\\ &\quad -2\left\{u_0+u_1+u_2+\cdots+\right\}+1\\ &= \sum_{s=0}^{\infty}\left\{(s+1)^2-s^2\right\}u_s-2\sum_{s=0}^{\infty}u_s+1\\ &= \sum_{s=0}^{\infty}(2\,s+1)\,u_s-2\sum_{s=0}^{\infty}u_s+1\\ &= 2\sum_{s=0}^{\infty}s\,u_s-\sum_{s=0}^{\infty}u_s+1. \end{aligned}\tag{67}$$

B. Form of $\{\sigma^{(p)}\}^2$

From (65) - (67), we have

$$\begin{aligned}
\left\{\sigma^{(p)}\right\}^2 &\triangleq E\left[\left\{X_\kappa^{(p)}\right\}^2\right] - \left\{E\left[X_\kappa^{(p)}\right]\right\}^2 \\
&= \sum_{r=1}^{\infty} \pi^{(\mathrm{D}_r)}\, {\ell_d}^2 + \sum_{s=1}^{\infty} \pi^{(\mathrm{E}_r)}\, \frac{1}{u_{s-1} - u_s} \int_{(s-1)\,\ell_d}^{s\,\ell_d} (x - (s-1)\,\ell_d)^2 \; dF^{(m)}(x) \\
&\quad - \left\{\ell^{(p)}\right\}^2 \\
&= \Delta^{-1}\, {\ell_d}^2\, (\Delta - 1) && \text{(from (27) and (29))} \\
&\quad + \Delta^{-1} \sum_{s=1}^{\infty} \int_{(s-1)\ell_d}^{s\,\ell_d} x^2\, dF^{(m)}(x) \\
&\quad - 2\,\Delta^{-1}\, \ell_d \sum_{s=1}^{\infty} (s-1) \int_{(s-1)\,\ell_d}^{s\,\ell_d} x\, dF^{(m)}(x) \\
&\quad + \Delta^{-1}\, {\ell_d}^2 \sum_{s=1}^{\infty} \int_{(s-1)\,\ell_d}^{s\,\ell_d} (s-1)^2 \; dF^{(m)}(x) - \left\{\ell^{(p)}\right\}^2 \\
&= {\ell_d}^2 \left(1 - \Delta^{-1}\right) + \Delta^{-1} \int_0^{\infty} x^2\, dF^{(m)}(x) - 2\,\Delta^{-1}\, \ell_d \sum_{s=1}^{\infty} (s-1)\,(v_{s-1} - v_s) \\
&\quad + \Delta^{-1}\, {\ell_d}^2 \sum_{s=1}^{\infty} (s-1)^2 \; (u_{s-1} - u_s) - \left\{\ell^{(p)}\right\}^2 \\
&= {\ell_d}^2 \left(1 - \Delta^{-1}\right) \\
&\quad + \Delta^{-1} \left(\left\{\sigma^{(m)}\right\}^2 + \left\{\ell^{(m)}\right\}^2\right) && \text{(from (65))} \\
&\quad - 2\,\Delta^{-1}\, \ell_d \sum_{s=0}^{\infty} v_s + 2\,\Delta^{-1}\, \ell_d\, \ell^{(m)} && \text{(from (66))} \\
&\quad + 2\,\Delta^{-1}\, {\ell_d}^2 \sum_{s=1}^{\infty} s\, u_s - \Delta^{-1}\, {\ell_d}^2 \sum_{s=0}^{\infty} u_s + \Delta^{-1}\, {\ell_d}^2 && \text{(from (67))} \\
&\quad - \left\{\ell^{(p)}\right\}^2 \\
&= \Delta^{-1} \left(\left\{\sigma^{(m)}\right\}^2 + \left\{\ell^{(m)}\right\}^2\right) - 2\,\Delta^{-1}\, \ell_d \sum_{s=0}^{\infty} v_s + 2\,\Delta^{-1}\, \ell_d\, \ell^{(m)} \\
&\quad + 2\,\Delta^{-1}\, {\ell_d}^2 \sum_{s=1}^{\infty} s\, u_s - \left\{\ell^{(p)}\right\}^2 .
\end{aligned} \tag{68}$$

Note that (68) agrees with the result reported in [8].

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