Bing doubling and the colored Jones polynomial

Sakie Suzuki

RIMS

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Introduction

Results

Motivation: Quantum invariants vs Topology

Motivation: Quantum invariants vs Topology

Classical Knot Theory

Motivation: Quantum invariants vs Topology

Classical Knot Theory

$\Uparrow$

Topology

Motivation: Quantum invariants vs Topology

Classical Knot Theory

Quantum Topology

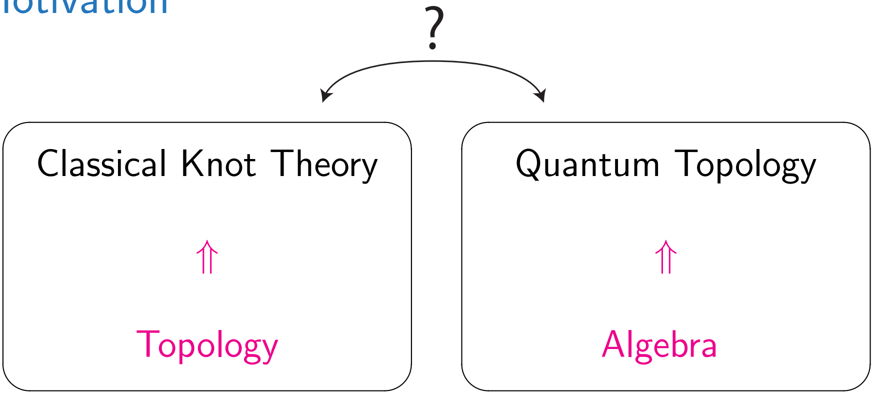
$\Uparrow$

Topology

Motivation: Quantum invariants vs Topology

Classical Knot Theory	Quantum Topology
$\Uparrow$	$\Uparrow$
Topology	Algebra

Motivation



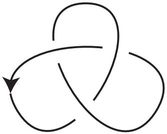
Motivation

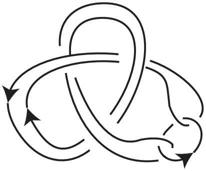
?

Classical Knot Theory	Quantum Topology
$\Uparrow$	$\Uparrow$
Topology	Algebra

Bong doubling $\longrightarrow$?

Bing doubling

$K =$ 

$B(K) =$ 

Quantum invariants

Links	ZHS
Kontsevich inv.	LMO inv.
Universal sl_2 inv.	Unified WRT inv.
Colored Jones poly.	WRT inv.

Quantum invariants

Links

Kontsevich inv.

Universal sl_2 inv.

Colored Jones poly.

ZHS

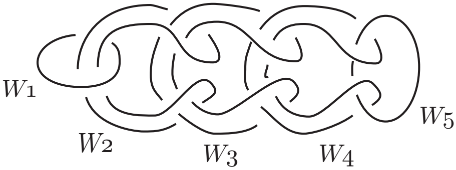
LMO inv.

Unified WRT inv.

WRT inv.

Colored Jones polynomial

$$W_1, \ldots, W_5 \in \mathrm{Mod}_f(U_h(sl_2)).$$



$$J_{L;W_1,\ldots,W_5} \in \mathbb{Z}[q^{1/4}, q^{-1/4}]$$

- Skein relation
- Operator invariant

Notation

For $i \in \mathbb{Z}, n \geq 0$,

$$\{i\} = q^{\frac{i}{2}} - q^{-\frac{i}{2}},$$
$$\{n\}! = \{n\}\{n-1\}\cdots\{1\},$$
$$\begin{bmatrix} i \\ n \end{bmatrix} = \{i\}\{i-1\}\cdots\{i-n+1\}/\{n\}!.$$

For $l \geq 0$,

$$P_l' = \frac{1}{\{l\}!}\prod_{i=0}^{l-1}(V_2 - q^{i+\frac{1}{2}} - q^{-i-\frac{1}{2}})$$
$$\in \mathrm{Span}_{\mathbb{Q}(q^{\frac{1}{2}})}\{V_m : m\text{-dim. irr. rep.} \mid m \geq 1\}.$$

Results for the colored Jones polynomial

Theorem (S)

Let K be a knot with 0-framing. For $i, j \geq 0$, we have

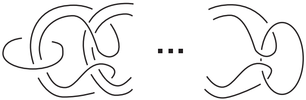
$$J_{B(K);P'_i,P'_j} = \sum_{l \geq 0} a_{i,j}^{(l)} J_{K;P'_l},$$

where

$$a_{i,j}^{(l)} = \delta_{i,j}(-1)^i \frac{\{2i+1\}!\{l\}!}{\{2l+1\}!}\lambda_{l,i},$$

$$\lambda_{l,i} = \sum_{k=0}^{l} (-1)^k \begin{bmatrix} 2l+1 \\ k \end{bmatrix} \begin{bmatrix} 2l+i-2k+1 \\ 2i+1 \end{bmatrix}.$$

Example

$M_n =$ 

$$J_{M_n;P'_1,\dots,P'_1} = (-1)^n q^{-2n+4}\Phi_1^{n-2}\Phi_2^{n-2}\Phi_3\Phi_4^{n-3}$$

$\Phi_m \in \mathbb{Z}[q]$: m-th cyclotomic polynomial

The unified WRT invariant

Set

$$\omega^{\pm 1} = \sum_{l=0}^{\infty} (\pm 1)^l q^{\pm \frac{1}{4} l(l+3)} P'_l.$$

$L = L_1 \cup \cdots \cup L_n$: link with framings $\epsilon_1, \ldots, \epsilon_n \in \{\pm 1\}$
$M = S^3_L$: ZHS obtained by surgery along L in S^3

Definition (Habiro)

$$J_M = J_{L^0; \omega^{-\epsilon_1}, \ldots, \omega^{-\epsilon_n}} \in \widehat{\mathbb{Z}[q]}.$$

Results for the unified WRT invariant

Theorem (S)

Let K be a knot with 0-framing.
Let M be an ZHS obtained by surgery along $B(K) \subset S^3$ with ± 1 framing.
We have

$$J_M - 1 \in \Phi_1^2 \Phi_2^2 \Phi_3 \Phi_4 \Phi_6 \widehat{\mathbb{Z}[q]}.$$

ありがとうございました。