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A Note on the Undecidability of Quantified Announcements

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# A Note on the Undecidability of Quantified Announcements

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**Abstract**

In [2], there is an error in the proof of the undecidability of group announcement logic (GAL). The purpose of this note is to correct this error. We show that when there are two or more agents in the language, the satisfiability problem is undecidable for GAL.

## 1 Introduction

Public announcement logic (PAL) is one of the well-known logics in the study of dynamic epistemic logic (see [3]). There are several logics with quantification over the announcement of PAL, for example, arbitrary public announcement logic (APAL), group announcement logic (GAL) and coalition announcement logic (CAL). Especially, it is shown that the satisfiability problem of GAL of 5 agents or more is undecidable in [1].

In "The Undecidability of Quantified Announcements" [2] T. Ågotnes, H. van Ditmarsch, and T. French demonstrate that the satisfiability problem of multiple agents APAL, GAL, and CAL is undecidable. However, there is an error in the proof of the undecidability of GAL. The purpose of this note is to correct this error. We show that when there are two or more agents in the language, the satisfiability problem is undecidable for GAL. In [2], it is mentioned that the satisfiability problem of GAL is decidable when the language contains only one agent. This and our result imply that two is the minimal number of agents that makes the problem undecidable.

In Section 2 we give a counterexample to a lemma in [2]. In Section 3 we give a new theorem on confluence. In Section 4 we show that the tiling problem is reducible to the satisfiability problem of GAL. All notations and definitions in this paper follow [2].

## 2 Counterexample of [2, Lemma 5.2]

The following formulas and a proposition are given in [2].

$$\begin{aligned} c_{\mathrm{ga}}(X) =& X \to [\mathfrak{s}]\,(K_{\mathfrak{s}}(r \to K_{\mathfrak{e}}(l \to K_{\mathfrak{s}}(u \to K_{\mathfrak{e}}(d \\ &\to K_{\mathfrak{s}}(l \to K_{\mathfrak{e}}(r \to K_{\mathfrak{s}}(d \to K_{\mathfrak{e}}(u \to \hat{K}_{\mathfrak{s}}X))))))))). \end{aligned}$$

$$cyc_{\mathrm{ga}} = c_{\mathrm{ga}}(\heartsuit) \wedge c_{\mathrm{ga}}(\clubsuit) \wedge c_{\mathrm{ga}}(\diamondsuit) \wedge c_{\mathrm{ga}}(\spadesuit).$$

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$$t_{\mathrm{ga}}(X,Y,Z) = X \to \bigwedge \begin{bmatrix} [\mathfrak{e}]\,(K_{\mathfrak{s}}(r \to K_{\mathfrak{e}}(l \to \hat{K}_{\mathfrak{s}}Y))) \\ [\mathfrak{e}]\,(K_{\mathfrak{s}}(u \to K_{\mathfrak{e}}(d \to \hat{K}_{\mathfrak{s}}Z))) \\ [\mathfrak{e}]\,(K_{\mathfrak{s}}(l \to K_{\mathfrak{e}}(r \to \hat{K}_{\mathfrak{s}}Y))) \\ [\mathfrak{e}]\,(K_{\mathfrak{s}}(d \to K_{\mathfrak{e}}(u \to \hat{K}_{\mathfrak{s}}Z))) \end{bmatrix}.$$

$$ck_{\mathrm{ga}} = t_{\mathrm{ga}}(\heartsuit, \clubsuit, \spadesuit) \wedge t_{\mathrm{ga}}(\clubsuit, \heartsuit, \diamondsuit) \wedge t_{\mathrm{ga}}(\diamondsuit, \spadesuit, \clubsuit) \wedge t_{\mathrm{ga}}(\spadesuit, \diamondsuit, \heartsuit).$$

$$CB_{\mathrm{ga}} = K_{\mathfrak{e}}K_{\mathfrak{s}}(\mathit{local} \wedge \mathit{cyc}_{\mathrm{ga}} \wedge ck_{\mathrm{ga}}).$$

**Proposition 2.1** ([2, Lemma 5.3(2)])**.** *Suppose that $M = (S, \sim, V), s \in S$ and $M_s \models CB_{\mathrm{ga}} \wedge \mathit{card}$. Then:*

- *For all $n \in \mathbb{N}$, for all $t \in sR(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}; l?; \mathfrak{e}; r?; \mathfrak{s}; d?; \mathfrak{e}; u?)$, there is some $u \sim_{\mathfrak{s}} t$ such that $u \in \|s\|^{\Pi}_{n}$.*

We show a counterexample to Proposition 2.1. Let $\Gamma = ((c_0, c_0, c_0, c_0), (c_1, c_0, c_1, c_0))$. We call $c_0$ white, and $c_1$ red. The structure of the counterexample is represented in Figure 1. It is made up of two checkerboards, one is formed only white squares, and the other is formed only red and white squares. The central nodes are labeled $c_0$. Note that agent $\mathfrak{e}$ is not able to distinguish between $t_1$ and $t'_2$. We confirm that the constructed model satisfies $M_s \models CB_{\mathrm{ga}} \wedge \mathit{card}$. In particular, $M_s \models c_{\mathrm{ga}}(\heartsuit)$ is supported by the following discussion. Let $\psi' \in \mathcal{L}_{el}$ be arbitrary, $\psi = K_{\mathfrak{s}}\psi'$ where $M_s \models \psi$. $M_{t_1} \models \psi$ since $t_1 \sim_{\mathfrak{s}} s$. There are three cases to consider:

Case 1: $M_{t'_2} \not\models \psi$ holds. Therefore $\{t \in S^{\psi} \mid t \sim_{\mathfrak{e}} t_1 \wedge t \in V(l)\}$ is empty or $\{t_2\}$.

Case 2: $M_{t'_2} \models \psi$ and there exists $v \in \{t'_3, t'_4, \ldots, t'_7, t'\}$ such that $M_v \not\models \psi$. Suppose $M_{t'_4} \not\models \psi$. $\{t \in S^{\psi} \mid t \sim_{\mathfrak{e}} t'_3 \wedge t \in V(d)\}$ is empty. The others are virtually identical to the $v = t'_4$ case.

Case 3: $M_{t'_2} \models \psi$ and for all $v \in \{t'_3, t'_4, \ldots, t'_7, t'\}$, $M_v \models \psi$. Since $\psi = K_{\mathfrak{s}}\psi'$, we have $M^{\psi}_{t'} \models \hat{K}_{\mathfrak{s}}\heartsuit$.

Based on the above, for all $\psi \in \mathcal{L}^{\mathfrak{s}}_{el}$,

$$M^{\psi}_{s} \models K_{\mathfrak{s}}(r \to K_{\mathfrak{e}}(l \to K_{\mathfrak{s}}(u \to K_{\mathfrak{e}}(d \to K_{\mathfrak{s}}(l \to K_{\mathfrak{e}}(r \to K_{\mathfrak{s}}(d \to K_{\mathfrak{e}}(u \to \hat{K}_{\mathfrak{s}}\heartsuit)))))))).$$

Therefore $M_s \models c_{\mathrm{ga}}(\heartsuit)$. Moreover for all $u \sim_{\mathfrak{s}} t'$ such that $M_s \models K_{\mathfrak{s}}(u \to c_0)$ and $M_u \not\models K_{\mathfrak{s}}(u \to c_0)$. We apply [2, Lemma 4.4] to $s$ and $u$, so $u \notin \|s\|_2$.

# 3 Introducing confluence instead of cycles

The purpose of this section is to show an alternative lemma to [2, Lemma 5.2]. In [2, Section 5.2], the formula $\mathit{cyc}_{\mathrm{ga}}$ which means cycle was given. In this section, instead of it, we give a formula $\mathit{con}_{\mathrm{ga}}$ that means confluence. For an arbitrary square, $\mathit{con}_{\mathrm{ga}}$ expresses the relationship between the up of right of it and the right of up of it. The properties are formalized as follows:

$$c^{*}_{\mathrm{ga}}(X,Y) = X \to [\mathfrak{s}](\hat{K}_{\mathfrak{s}}(u \wedge \hat{K}_{\mathfrak{e}}(d \wedge \hat{K}_{\mathfrak{s}}(r \wedge K_{\mathfrak{e}}K_{\mathfrak{s}}\neg Y))) \to K_{\mathfrak{s}}(r \to K_{\mathfrak{e}}(l \to K_{\mathfrak{s}}(u \to K_{\mathfrak{e}}(d \to K_{\mathfrak{s}}\neg Y))))).$$

$$\mathit{con}_{\mathrm{ga}} = c^{*}_{\mathrm{ga}}(\heartsuit, \diamondsuit) \wedge c^{*}_{\mathrm{ga}}(\diamondsuit, \heartsuit) \wedge c^{*}_{\mathrm{ga}}(\spadesuit, \clubsuit) \wedge c^{*}_{\mathrm{ga}}(\clubsuit, \spadesuit).$$

$$CB^{*}_{\mathrm{ga}} = K_{\mathfrak{e}}K_{\mathfrak{s}}(\mathit{local} \wedge \mathit{con}_{\mathrm{ga}} \wedge ck_{\mathrm{ga}}).$$

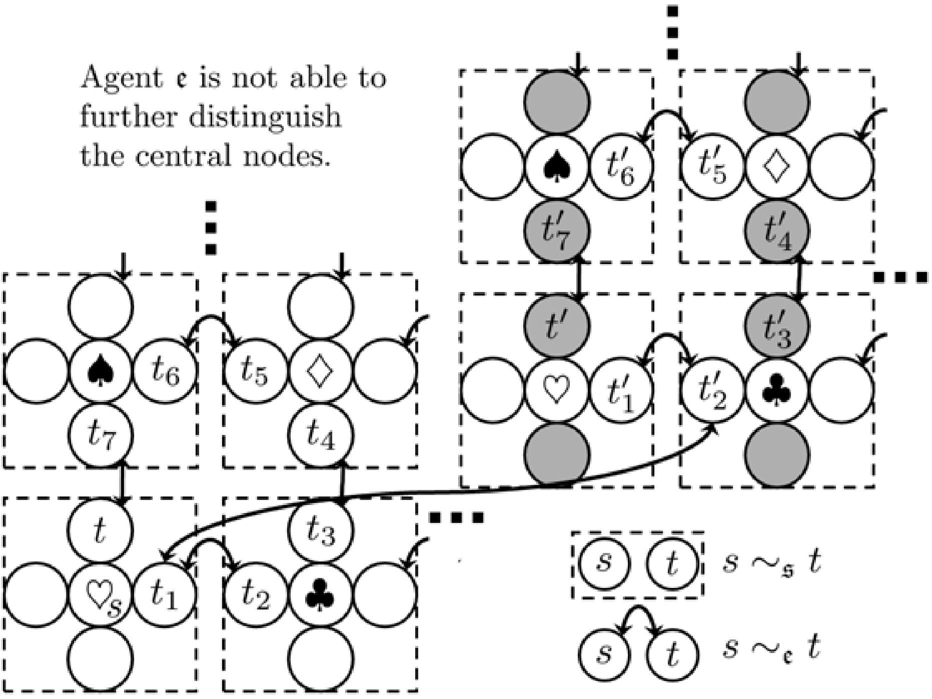


Figure 1: A counterexample model

**Lemma 3.1** (cf. [2, Lemma 5.3(1)]). *Suppose that* $M = (S, \sim, V), s \in S$, *and* $M_s \models K_{\mathfrak{e}} K_{\mathfrak{s}}(local \wedge ck_{\mathrm{ga}}) \wedge card$. *Let* $U = sR(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}) \cup sR(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}) \cup sR(\mathfrak{s}; l?; \mathfrak{e}; r?; \mathfrak{s}) \cup sR(\mathfrak{s}; d?; \mathfrak{e}; u?; \mathfrak{s})$. *Then:*

- *For all* $n \in \mathbb{N}$, *for all* $t' \in U$, *there is some* $t \sim_{\mathfrak{s}} t'$, *and some* $u \sim_{\mathfrak{e}} s$ *such that* $u \in \|t\|^{\Pi}_n$.

*Proof.* We follow the proof of [2, Lemma 5.3(1)] except that $\psi$ is not $l \vee r \vee K_{\mathfrak{e}}(\bigvee_{a \leq m} \phi_a)$ but $K_{\mathfrak{e}}(l \vee r \vee \bigvee_{a \leq m} \phi_a)$.

□

**Lemma 3.2.** *Suppose that* $M = (S, \sim, V), s \in S$ *and* $M_s \models CB^*_{\mathrm{ga}} \wedge card$. *Let* $U_1 = sR(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}; r?)$, $U_2 = sR(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?; \mathfrak{e}; d?)$. *Then:*

- *For all* $n \in \mathbb{N}$, *for all* $t'_1 \in U_1$, *for all* $t'_2 \in U_2$, *there is some* $t_1 \in t'_1 R(\mathfrak{e}; l?)$ *and some* $t_2 \sim_{\mathfrak{s}} t'_2$ *such that* $t_1 \in \|t_2\|^{\Pi}_n$.

*Proof.* We consider any two chains for worlds

$$s \sim_{\mathfrak{s}} w_1 \sim_{\mathfrak{e}} w_2 \sim_{\mathfrak{s}} t'_1 \sim_{\mathfrak{e}} t_1,$$

$$s \sim_{\mathfrak{s}} v_1 \sim_{\mathfrak{e}} v_2 \sim_{\mathfrak{s}} v_3 \sim_{\mathfrak{e}} t'_2.$$

Where $w_1, v_3 \in V(u), t'_1, v_2 \in V(l), w_2, t'_2 \in V(d), w_3, v_1 \in V(r)$. Each world in these chains satisfies the formula *local*, since Lemma 3.1. (Detailed proof is similar to the proof described in [2, Lemma 5.2(2)].)

Let us suppose that $M_s \models \heartsuit$. The construction we have applied is depicted in Figure 2. Suppose, for contradiction, that there is some $n \in \mathbb{N}$, some $t'_1 \in U_1$, and some $t'_2 \in U_2$ such that for all $t_1 \in t'_1 R(\mathfrak{e}; l?)$ and for all $t_2 \sim_{\mathfrak{s}} t'_2$, $t_1 \notin \|t_2\|^{\Pi}_n$.

From [2, Lemma 4.3] for every $n$, there exists a set of formulas, $\phi_0, \ldots, \phi_m$ such that

(1) For all $u \sim_{\mathfrak{s}} t'_2$ there exists $a \leq m$ such that $M_u \models \phi_a$.

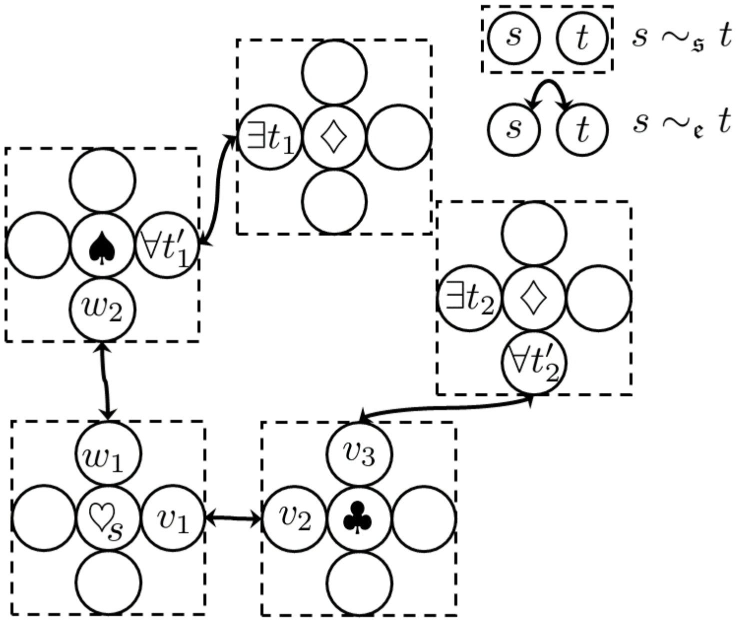


Figure 2: A representation of Lemma 3.2

(2) For all $a \leq m$, there exists $u \sim_{\mathfrak{s}} t'_2$ such that $M_u \models \phi_a$.

(3) For all $a \leq m$, for all $u \sim_{\mathfrak{s}} t'_2$ where $M_u \models \phi_a$, for all $t \in S$, $M_t \models \phi_a$ if and only if $t \in \|u\|_n$.

It follows that $M_{t'_2} \models K_{\mathfrak{s}} \bigvee_{a \leq m} \phi_a$ and for all $t_1 \in t'_1 R(\mathfrak{e}; l?)$, $M_{t_1} \models \neg \bigvee_{a \leq m} \phi_a$ by assumption. Let $\psi = K_{\mathfrak{s}}(\hat{K}_{\mathfrak{s}} \diamondsuit \rightarrow \bigvee_{a \leq m} \phi_a)$. Then:

(a) For all $s' \in \{s, v_1, v_2, v_3, w_1, w_2, t'_1\}$, $M_{s'} \models \psi$. Since $M_{s'} \models local$, we have $M_{s'} \models K_{\mathfrak{s}} \neg \hat{K}_{\mathfrak{s}} \diamondsuit$.

(b) $M_{t'_2} \models \psi$ holds, since $M_{t'_2} \models K_{\mathfrak{s}} \bigvee_{a \leq m} \phi_a$.

(c) There exists $t^*_2 \sim_{\mathfrak{s}} t'_2$ such that $M_{t^*_2} \models \diamondsuit$, because $M_{t'_2} \models local$. $M_{t^*_2} \models K_{\mathfrak{s}} \bigvee_{a \leq m} \phi_a$, since $M_{t'_2} \models K_{\mathfrak{s}} \bigvee_{a \leq m} \phi_a$.

(d) For all $t_1 \in t'_1 R(\mathfrak{e}; l)$, if $M_{t_1} \models \psi$, then $M_{t_1} \models K_{\mathfrak{s}} \neg \diamondsuit$. By assumption, we have $M_{t_1} \models \neg \bigvee_{a \leq m} \phi_a$. If $M_{t_1} \models \psi$, then $M_{t_1} \models K_{\mathfrak{s}} \neg \diamondsuit$.

(a),(b),(c),(d) and for all $t_1 \sim_{\mathfrak{e}} t'_1$ where $t_1 \notin V(l)$, we have $M_{t_1} \models K_{\mathfrak{s}} \neg \diamondsuit$. Therefore

$$M^{\psi}_s \models \hat{K}_{\mathfrak{s}}(u \wedge \hat{K}_{\mathfrak{e}}(d \wedge \hat{K}_{\mathfrak{s}}(r \wedge K_{\mathfrak{e}} K_{\mathfrak{s}} \neg \diamondsuit))) \wedge \hat{K}_{\mathfrak{s}}(r \wedge \hat{K}_{\mathfrak{e}}(l \wedge \hat{K}_{\mathfrak{s}}(u \wedge \hat{K}_{\mathfrak{e}}(d \wedge \hat{K}_{\mathfrak{s}} \diamondsuit)))).$$

This is inconsistent with $M_s \models con_{\mathrm{ga}}$.

It can be shown similarly for cases other than $M_s \models \heartsuit$.

□

**Corollary 3.3** (cf. [2, Lemma 6.1]). *Suppose that $M = (S, \sim, V)$, $s \in S$ and $M_s \models CB^*_{\mathrm{ga}} \wedge card$. Let $U = sR(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}) \cup sR(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}) \cup sR(\mathfrak{s}; l?; \mathfrak{e}; r?; \mathfrak{s}) \cup sR(\mathfrak{s}; d?; \mathfrak{e}; u?; \mathfrak{s})$. Then, for all $s' \sim_{\mathfrak{e}} s$:*

1. *For all $n \in \mathbb{N}$, for all $t' \in U$, there is some $t \sim_{\mathfrak{s}} t'$, and some $u \sim_{\mathfrak{e}} s$ such that $u \in \|t\|^{\Pi}_n$.*

2. *For all $n \in \mathbb{N}$, for all $t'_1 \in s'R(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}; r?)$, for all $t_2 \in s'R(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$, there is some $t_1 \in t'_1 R(\mathfrak{e}; l?; \mathfrak{s})$ such that $t_1 \in \|t_2\|^{\Pi}_n$.*

*Proof.* 1. Follows from Lemma 3.1.

2. We note that $M_{s'} \models CB^*_{\text{ga}} \wedge card$, because $s' \sim_{\mathfrak{e}} s$. Let $t'_2 \in t_2 R(\mathfrak{s}; d) \cap s' R(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?\mathfrak{e}; d?)$. There is some $t^*_1 \in t'_1 R(\mathfrak{e}; l?)$, some $t^*_2 \sim_{\mathfrak{s}} t'_2$ such that $t^*_1 \in \|t^*_2\|_{n+1}$, since Lemma 3.2. Since $t_2 \sim_{\mathfrak{s}} t'_2 \sim_{\mathfrak{s}} t^*_2$, $t_2 \sim_{\mathfrak{s}} t^*_2$ holds. There is some $t_1 \sim_{\mathfrak{s}} t^*_1$ such that $t_1 \in \|t_2\|_n$, since the definition of bisimulation.

□

# 4 Undecidability

As with [2, Section 6], we show the undecidability of the satisfiability problem of GAL. We note that Corollary 3.3(2) is weaker than [2, Theorem 6.1(2)]. Therefore, proof of Lemma 4.1 is different from [2, Lemma 6.2] in some situations. (Especially for the case of $i \neq 0 \wedge j \neq 0$.)

**Lemma 4.1** (cf. [2, Lemma 6.2])**.** *Suppose $M_s \models SAT_\Gamma \wedge CB^*_{\text{ga}} \wedge \heartsuit$. Then $\Gamma$ can tile the plane.*

*Proof.* Given the model $M = (S, \sim, V)$, and the state $s$. Let $P_n = \{(i,j) \in \mathbb{N} \times \mathbb{N} \mid i + j \leq n\}$. We show that there exists a map $\tau_n : P_n \to S$ for each $n \in \mathbb{N}$ such that:

- $\tau_n(0,0) = s$.
- For all $i, j \in \mathbb{N}$, if $i + j \leq n$, then $\tau_n(i,j) \sim_{\mathfrak{e}} s$.
- For all $i, j \in \mathbb{N}$, if $n > i + j + 1$, then there exists $t \in \tau_n(i,j) R(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s})$ such that $\tau_n(i+1, j) \in \|t\|_{3(n-(i+j+1))}$.
- For all $i, j \in \mathbb{N}$, if $n > i + j + 1$, then there exists $t \in \tau_n(i,j) R(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$ such that $\tau_n(i, j+1) \in \|t\|_{3(n-(i+j+1))}$.

We show the existence of $\tau_n$ for each n by induction. Let $\tau_n(0,0) = s$. For arbitrary $(i,j) \in \mathbb{N} \times \mathbb{N}$, we assume that $\tau_n(i', j')$ has been defined for all $(i', j')$ where $i' + j' < i + j$. Let $m = n - (i + j)$. There are three inductive cases to consider:

1. $i = 0$: Detailed proof is equivalent to the proof described in [2, Lemma 6.2].
2. $j = 0$: Detailed proof is equivalent to the proof described in [2, Lemma 6.2].
3. $i \neq 0 \wedge j \neq 0$: We suppose that $\tau_n(i-1, j-1) = s'$, $\tau_n(i, j-1) = s_u$, $\tau_n(i-1, j) = s_r$.

   By the inductive hypothesis we have that

   (a) There exists $t_1 \in s' R(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$ such that $t_1 \in \|s_u\|_{3m+3}$.

   (b) There exists $t_2 \in s' R(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s})$ such that $t_2 \in \|s_r\|_{3m+3}$.

   The structure of this case is pictured in Figure 3. Since $s_r \sim_{\mathfrak{e}} s$ and Corollary 3.3(1), there is some $w \in s_r R(\mathfrak{s}; d?; \mathfrak{e}; d?; \mathfrak{s})$ and some $s^* \sim_{\mathfrak{e}} s_r$ such that $w \in \|s^*\|_{3m}$. Since $M_{s'} \models K_{\mathfrak{e}} K_{\mathfrak{s}} local$ and Corollary 3.3(1), for all $w^* \in s_r R(\mathfrak{s}; u?)$, $M_{w^*} \models local$. Therefore $s_r R(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$ is not empty. It is similarly shown that other worlds are not empty. Let $\tau_n(i,j) = s^*$. $s^* \sim_{\mathfrak{e}} s_u \sim_{\mathfrak{e}} s$ holds, hence $s^* \sim_{\mathfrak{e}} s$ holds. Since the definition of bisimulation, there is some $w' \in t_2 R(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$ such that $w \in \|w_2\|_{3m}$. We note $w' \in s' R(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s})$. By Corollary 3.3(2), for all $t'_1 \in t_1 R(\mathfrak{s}; r?)$, there is some $v' \in t'_1 R(\mathfrak{e}; l?; \mathfrak{s})$ such that $v' \in \|w'\|_{3m}$. Since $t_1 \in \|s_u\|_{3m+3}$, we can choose $v$ to satisfy $v' \in \|v\|_{3m}$. By the transitivity of bisimulation, $s^* \in \|v\|_{3m}$ holds.

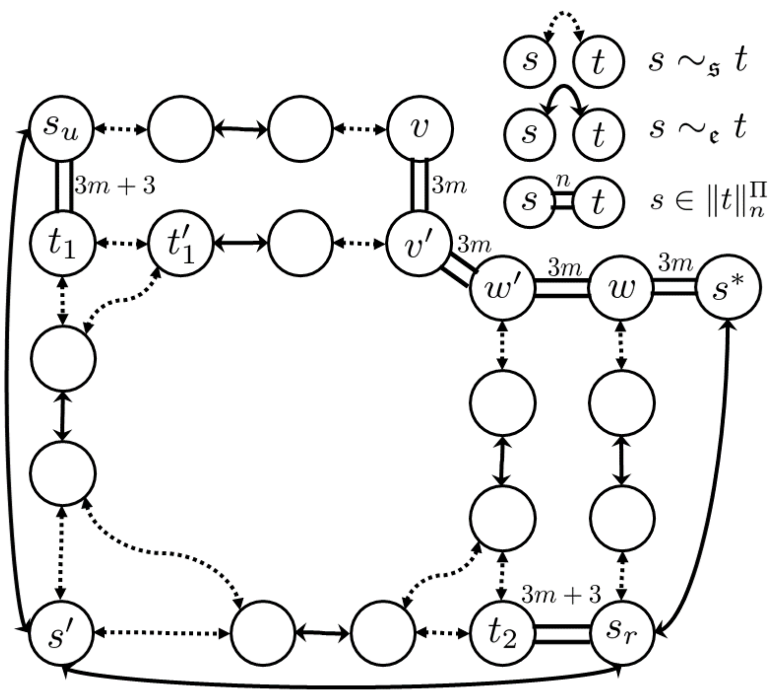


Figure 3: A representation of the final case of Lemma 4.1

Therefore, we can suppose the existence of the function $\tau_n$ for all $n \in \mathbb{N}$. We are able to show that $\Gamma$ can tile the plane, in the same way as [2, Theorem 6.2].

□

**Lemma 4.2** (cf. [2, Lemma 6.3])**.** *Suppose that* $\Gamma$ *can tile the plane. Then there exists model* $M = (S, \sim, V)$ *and some state* $s \in S$ *such that*

$$M_s \models SAT_\Gamma \wedge CB^*_{\text{ga}} \wedge \heartsuit.$$

*(*$SAT_\Gamma$ *is defined in [2, Section 5].)*

*Proof.* Suppose that $\Gamma$ is possible to tile the plane, there is a function $\lambda : \mathbb{N} \times \mathbb{N} \to \Gamma$ such that for all $(i, j) \in \mathbb{N} \times \mathbb{N}$. We build the model $M = (S, \sim, V)$ similarly to [2, Lemma 6.3]. (Let $\sim_{\mathfrak{s}} = R_{\mathfrak{s}}$ and $\sim_{\mathfrak{e}} = R_{\mathfrak{e}}$.)

In particular, we confirm $M_s \models K_{\mathfrak{e}} K_{\mathfrak{s}} con_{\text{ga}}$. (Other formulas follow [2, Lemma 6.3].) Let $s = (0, 0, mid)$. Then

- $M_s \models K_{\mathfrak{e}} K_{\mathfrak{s}} con_{\text{ga}}$.
  Suppose $t = (i, j, k) \in S$ and $t \in V(\heartsuit)$. We note that $\{(i+1, j+1, mid)\} = tR(\mathfrak{s}; u?; \mathfrak{e}; d?; \mathfrak{s}; r?; \mathfrak{e}; l?\mathfrak{s}; \diamondsuit?)$ and $\{(i+1, j+1, mid)\} = tR(\mathfrak{s}; r?; \mathfrak{e}; l?; \mathfrak{s}; u?; \mathfrak{e}; d?\mathfrak{s}; \diamondsuit?)$. Let $\psi$ (where $\psi = K_{\mathfrak{s}}\psi', \psi' \in \mathcal{L}_{el}$) is arbitrary. If $M_t^\psi \models (\hat{K}_{\mathfrak{s}}(u \wedge \hat{K}_{\mathfrak{e}}(d \wedge \hat{K}_{\mathfrak{s}}(r \wedge K_{\mathfrak{e}} K_{\mathfrak{s}} \neg \diamondsuit))))$, then $(i+1, j+1, mid) \notin S^\psi$. Therefore $M_t^\psi \models K_{\mathfrak{s}}(r \to K_{\mathfrak{e}}(l \to K_{\mathfrak{s}}(u \to K_{\mathfrak{e}}(d \to K_{\mathfrak{s}} \neg \diamondsuit))))$. Similar arguments can be given for other directions and suits so we have $M_t \models con_{\text{ga}}$. By the arbitrariness of $t$, $M_s \models K_{\mathfrak{e}} K_{\mathfrak{s}} con_{\text{ga}}$.

□

**Theorem 4.3** (cf. [2, Lemma 6.4])**.** *The satisfiability problem for GAL is undecidable, provided that there is more than one agent in the system.*

*Proof.* From Lemma 4.1 and Lemma 4.2, it is shown that the satisfiability problem for GAL is equivalent to a tiling problem. Therefore, when there are two or more agents in the language, the satisfiability problem is undecidable for GAL. □

# References

[1] Ågotnes, T., H. van Ditmarsch, and T. French, The Undecidability of Group Announcements, in *Proceedings of the 13th AAMAS*, 2014, pp. 893-900.

[2] Ågotnes, T., H. van Ditmarsch, and T. French, The Undecidability of Quantified Announcements, *Studia Logica* 104(4), 2016, pp. 597-640.

[3] van Ditmarsch, H., W. van der Hoek, and B. Kooi, Dynamic Epistemic Logic, *S*pringer, 2008.